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Parent Functions and Transformations

Unit 1 Lesson 5

Parent Functions and Transformations

Students will be able to:

Identify the effect on the graph of replacing

$f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$

for specific values of k (both positive and negative);

find the value of k given the graphs.

Parent Functions and Transformations

Key Vocabulary:

Parent function

Transformation

Translation

Dilation

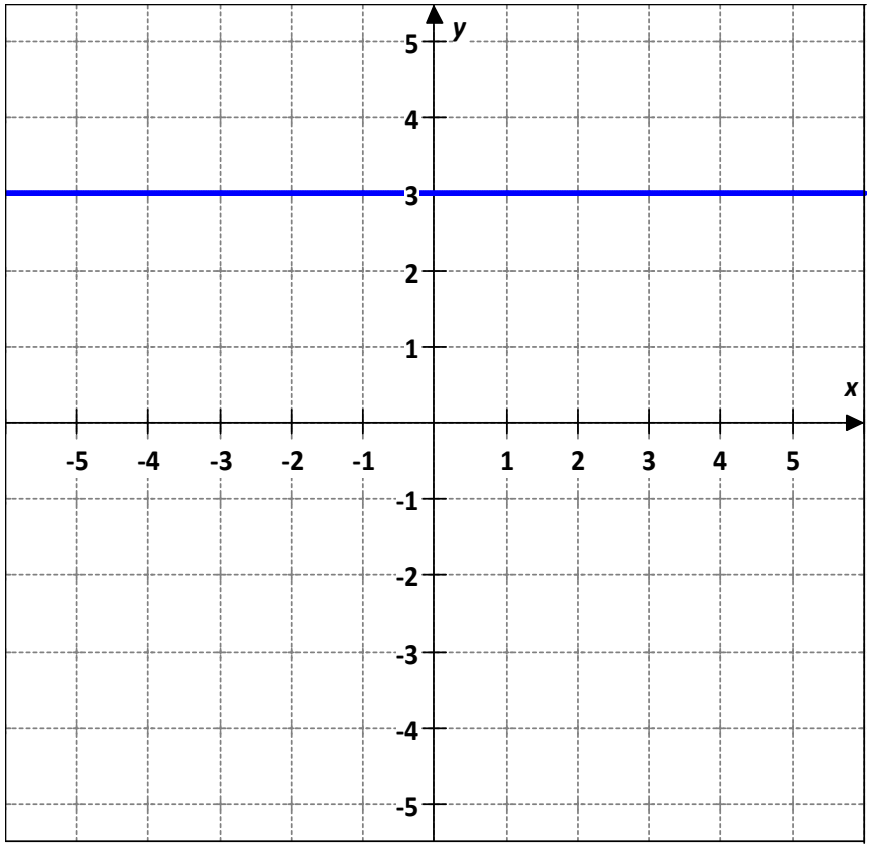
Parent Functions and Transformations

A family of functions is a group of functions with graphs that display one or more similar characteristics.

The Parent Function is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

Family - Constant Function

Graph:



Rule $f(x) = c$

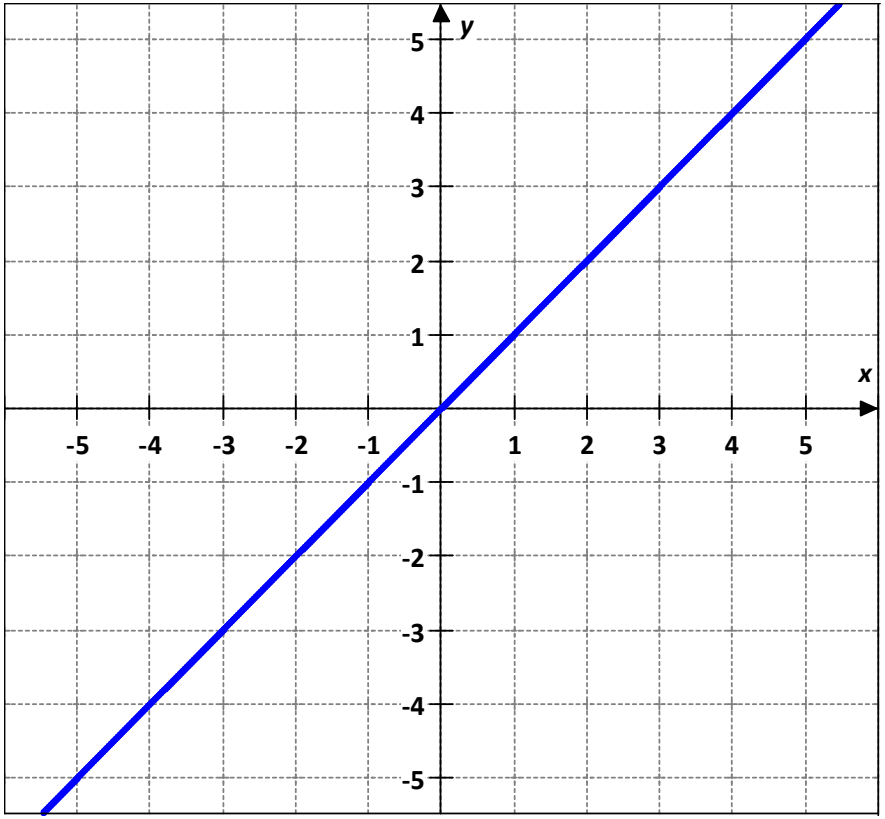
Domain $= (-\infty, \infty)$

Range $= [c]$

Parent Functions and Transformations

Family - Linear Function

Graph:



Rule $f(x) = x$

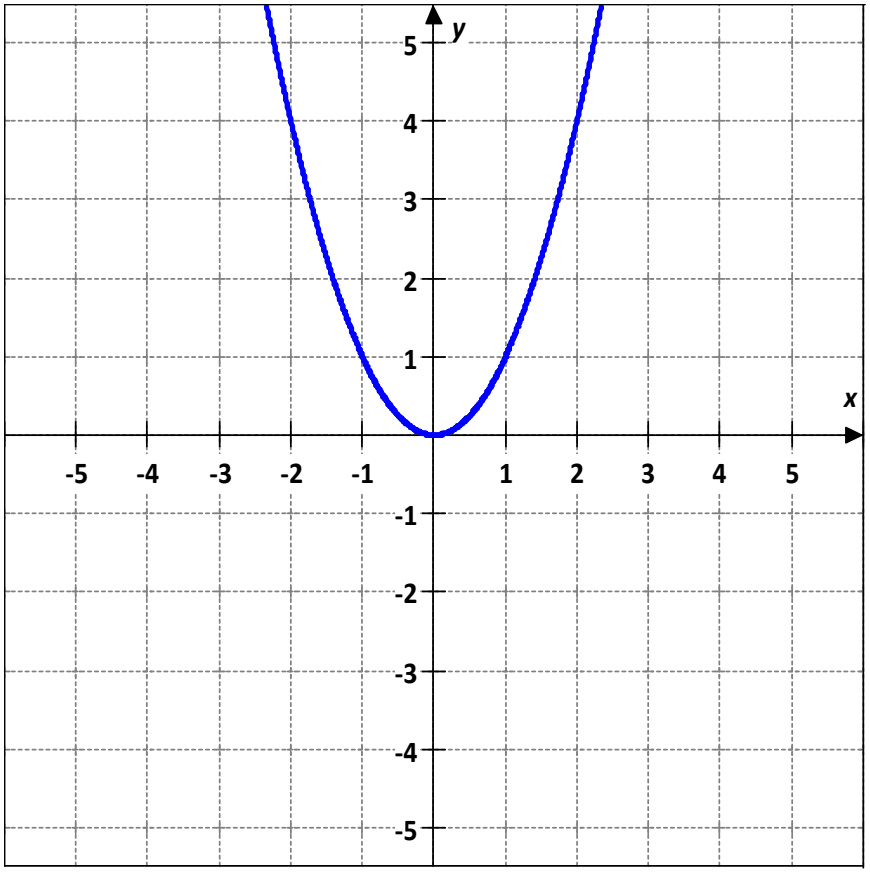
Domain $= (-\infty, \infty)$

Range $= (-\infty, \infty)$

Parent Functions and Transformations

Family - Quadratic Function

Graph:



Rule $f(x) = x^2$

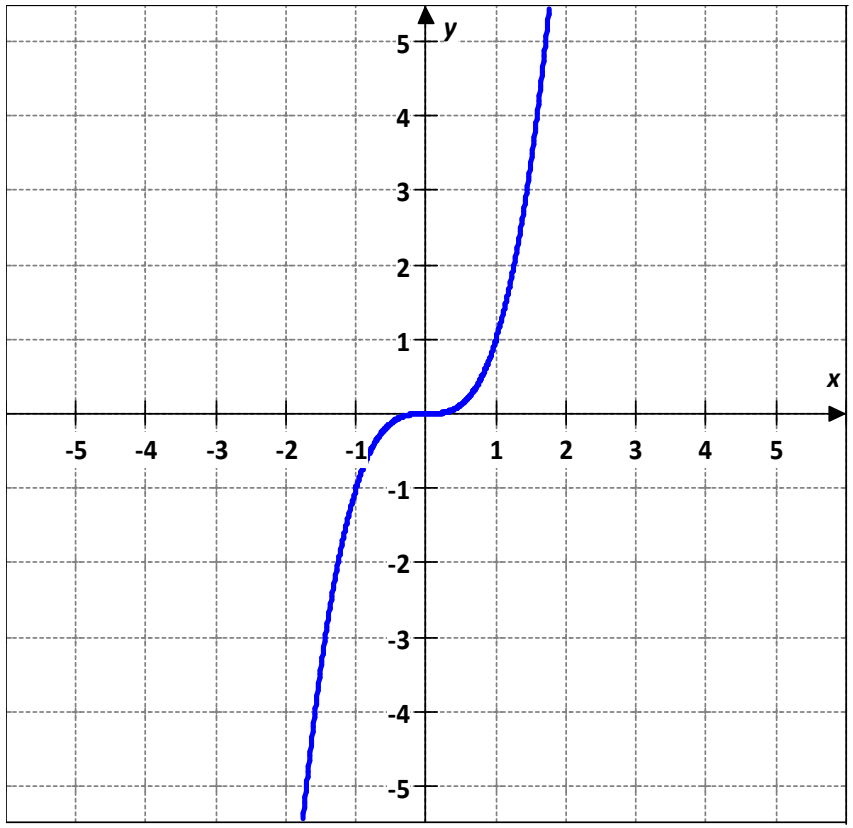
Domain $= (-\infty, \infty)$

Range $= [0, \infty)$

Parent Functions and Transformations

Family - Cubic Function

Graph:



Rule $f(x) = x^3$

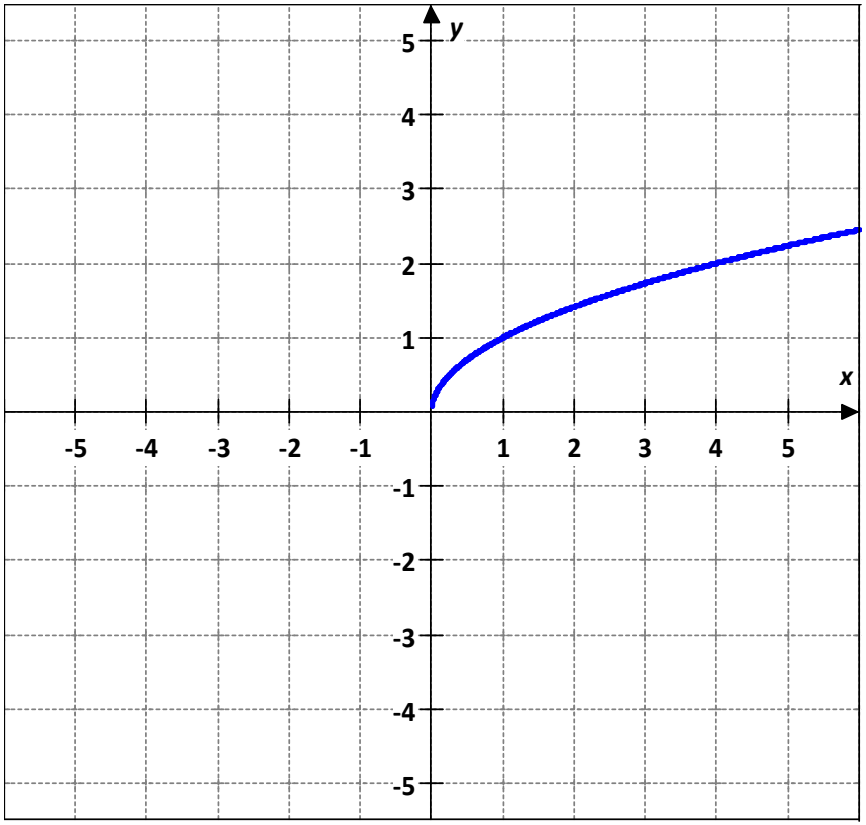
Domain $= (-\infty, \infty)$

Range $= (-\infty, \infty)$

Parent Functions and Transformations

Family - Square Root Function

Graph:



Rule $f(x) = \sqrt{x}$

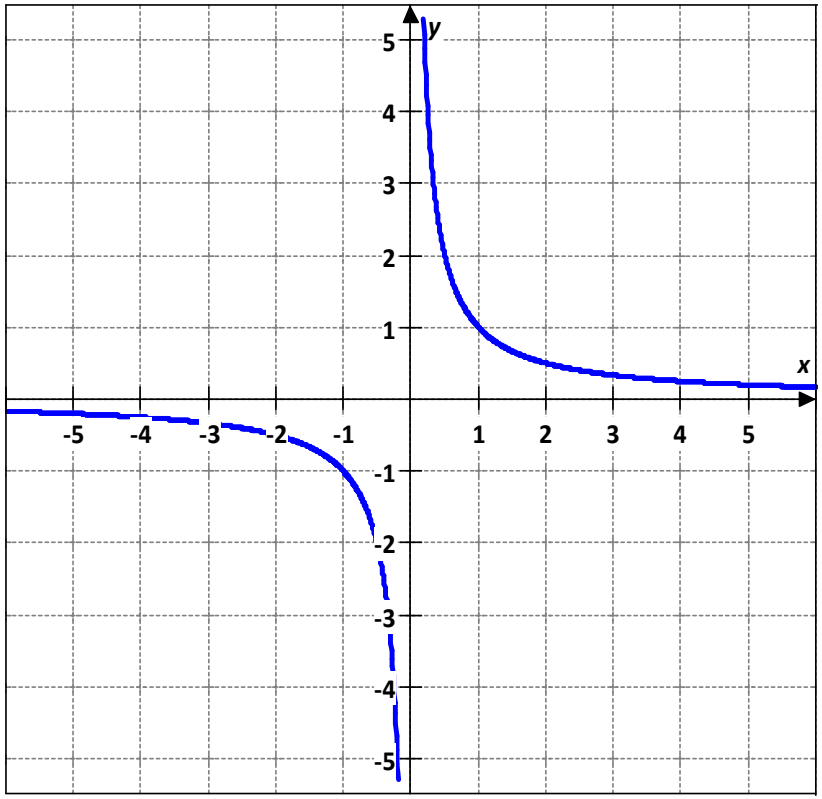
Domain $= [0, \infty)$

Range $= [0, \infty)$

Parent Functions and Transformations

Family - Reciprocal Function

Graph:



Rule $f(x) = \frac{1}{x}$

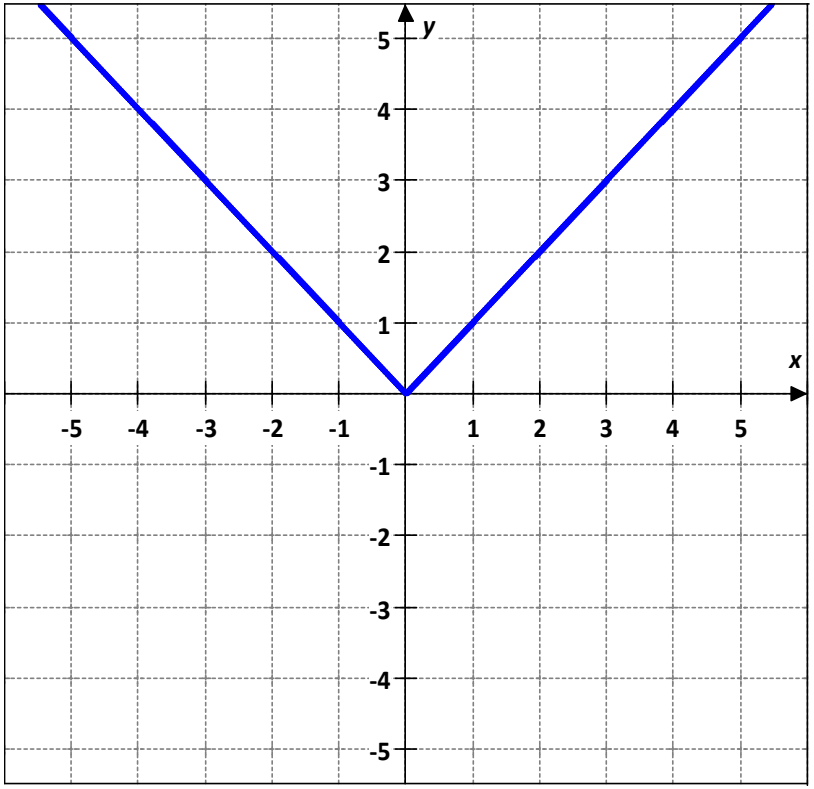
$D = (-\infty, 0) \cup (0, \infty)$

$R = (-\infty, 0) \cup (0, \infty)$

Parent Functions and Transformations

Family - Absolut Value Function

Graph:



Rule $f(x) = |x|$

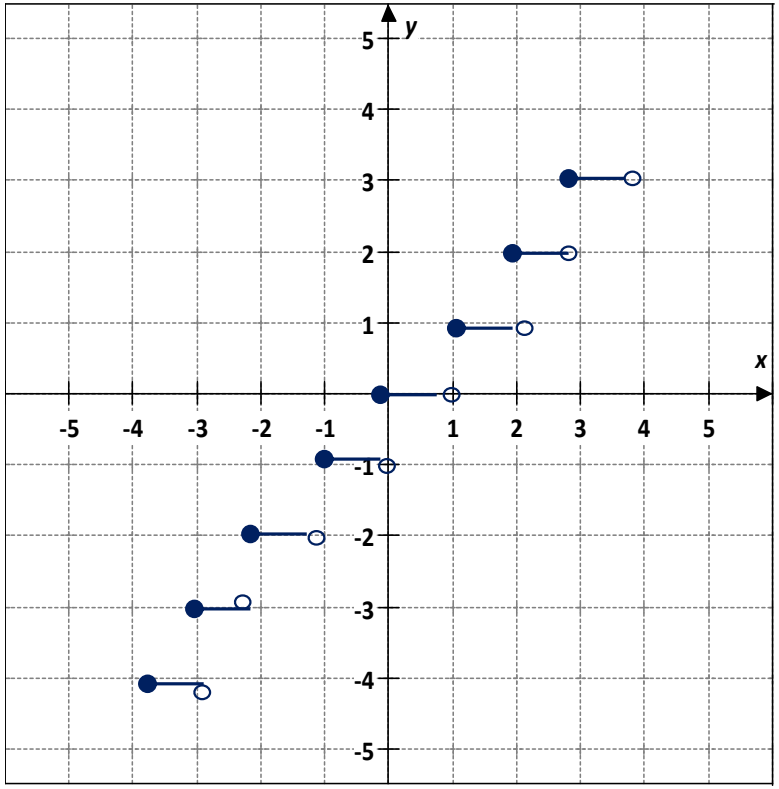
$$|x| = \begin{cases} -x & if \quad x < 0 \\ x & if \quad x \geq 0 \end{cases}$$

$D = (-\infty, \infty)$

$R = [0, \infty)$

Family - Greatest Integer Function

Graph:



Rule $f(x) = [\![x]\!]$

$D = (-\infty, \infty)$

$R =$ **All Integer**

Transformations

A change in the size or position of a figure or graph of the function is called a transformation.

Rigid transformations change only the position of the graph, leaving the size and shape unchanged.

Non rigid transformations distort the shape of the graph.

Parent Functions and Transformations

Rigid transformations

Vertical Translations

Appearance in Function	Transformation of Graph	Transformation of Point
$f(x) \rightarrow f(x) + a$	a *units up*	$(x, y) \rightarrow (x, y + a)$
$f(x) \rightarrow f(x) - a$	a *units down*	$(x, y) \rightarrow (x, y - a)$

Parent Functions and Transformations

Rigid transformations

Horizontal Translations

Appearance in Function	Transformation of Graph	Transformation of Point
$f(x) \rightarrow f(x-b)$	b *units right*	$(x, y) \rightarrow (x+b, y)$
$f(x) \rightarrow f(x+b)$	b *units left*	$(x, y) \rightarrow (x-b, y)$

Parent Functions and Transformations

Rigid transformations

Reflections in x-axes

Appearance in Function	Transformation of Graph	Transformation of Point
$\boldsymbol{f(x) \rightarrow -f(x)}$	$\boldsymbol{reflected\ in\ the\ x\ axis}$	$\boldsymbol{(x, y) \rightarrow (x, -y)}$

Parent Functions and Transformations

Rigid transformations

Reflections in y-axes

Appearance in Function	Transformation of Graph	Transformation of Point
$f(x) \rightarrow f(-x)$	$reflected\ in\ the\ \boldsymbol{y\ axis}$	$(x, y) \rightarrow (-x, y)$

Parent Functions and Transformations

Non rigid transformations

Vertical Dilations

Appearance in Function	Transformation of Graph	Transformation of Point
$f(x) \to cf(x)$ $c > 1$	*expanded vertically*	$(x, y) \to (x, cy)$
$f(x) \to cf(x)$ $0 < c < 1$	*compressed vertically*	$(x, y) \to (x, cy)$

Parent Functions and Transformations

Non rigid transformations

Horizontal Dilations

Appearance in Function	Transformation of Graph	Transformation of Point
$f(x) \to f(dx)$ $d > 1$	*compressed horizontally*	$(x, y) \to \left(\frac{x}{d}, y\right)$
$f(x) \to f(dx)$ $0 < d < 1$	*expanded horizontally*	$(x, y) \to \left(\frac{x}{d}, y\right)$

Sample Problem 1: **Identify the parent function and describe the transformations.**

a. $f(x) = (x - 1)^2$

Parent Functions and Transformations

Sample Problem 1: Identify the parent function and describe the transformations.

a. $f(x) = (x - 1)^2$

Parent : $f(x) = x^2$

Transformation: Translation 1 unit right

Sample Problem 1: **Identify the parent function and describe the transformations.**

b. $f(x) = x^3 - 5$

Sample Problem 1: **Identify the parent function and describe the transformations.**

b. $f(x) = x^3 - 5$

Parent : $f(x) = x^3$

Transformation: Translation 5 units down

Sample Problem 1: **Identify the parent function and describe the transformations.**

c. $f(x) = -|x + 4|$

Sample Problem 1: **Identify the parent function and describe the transformations.**

c. $f(x) = -|x + 4|$

Parent : $f(x) = |x|$

Transformation: Reflection in x-axis
Translation 4 units left

Sample Problem 1: **Identify the parent function and describe the transformations.**

d. $f(x) = 3x^2 + 7$

Sample Problem 1: Identify the parent function and describe the transformations.

d. $f(x) = 3x^2 + 7$

Parent : $f(x) = x^2$

Transformation: Expand vertically by a factor of 3
Translation 7 units up

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

a. **Quadratic - expanded horizontally by a factor of 2, translated 7 units up.**

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

a. **Quadratic - expanded horizontally by a factor of 2, translated 7 units up.**

$$f(x) = \frac{1}{2}x^2 + 7$$

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

b. Cubic - reflected over the x axis and translated 9 units down.

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

b. Cubic - reflected over the x axis and translated 9 units down.

$$f(x) = -x^3 - 9$$

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

c. Absolute value - translated 3 units up, translated 8 units' right.

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c. Absolute value - translated 3 units up, translated 8 units right.

$$f(x) = |x - 8| + 3$$

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

d. Reciprocal - translated 1 unit up.

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

d. Reciprocal - translated 1 unit up.

$$f(x) = \frac{1}{x} + 1$$

Sample Problem 3: **Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

a. $h(x) = 2(x-3)^2 - 2$

Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

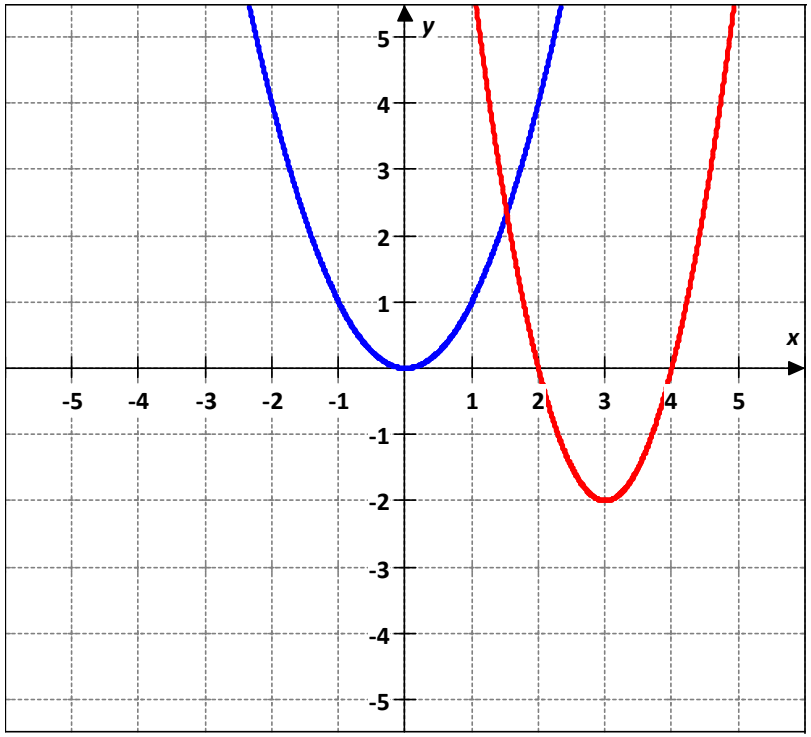
a. $h(x) = 2(x-3)^2 - 2$

$h(x) = 2(x-3)^2 - 2$ ⟶

Parent function $f(x) = x^2$ ⟶

Transformation:
Compressed horizontally by a factor of 2
Translated 2 units down
Translated 3 units right

$D = (-\infty, \infty)$ $R = (-2, \infty)$



Sample Problem 3: **Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

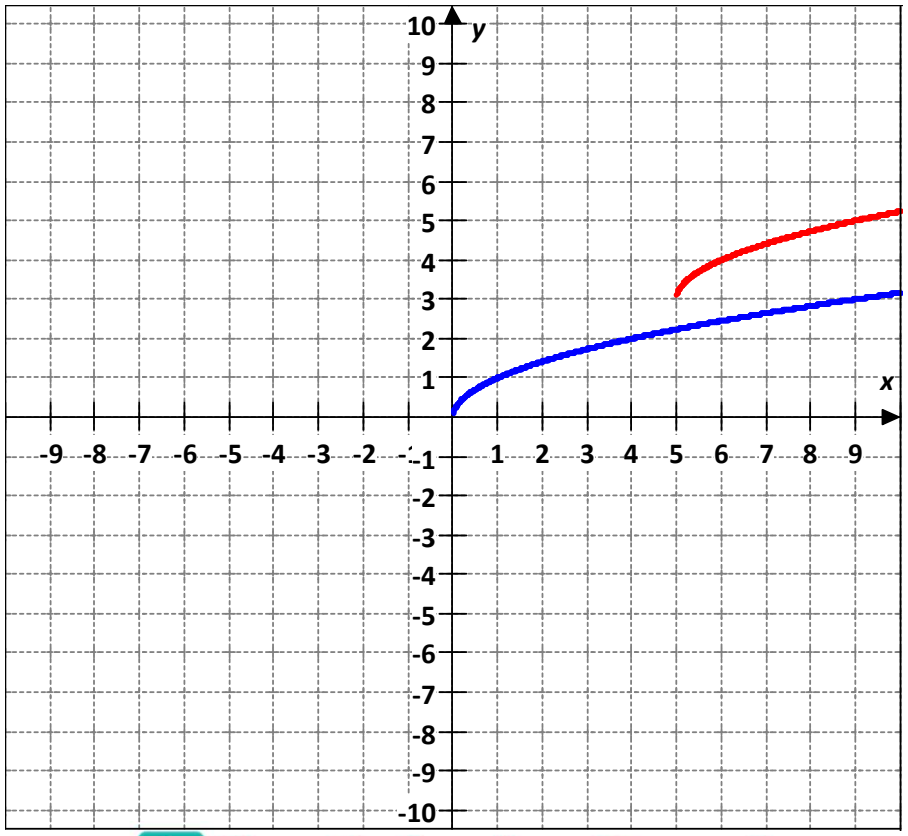
b. $h(x) = \sqrt{x - 5} + 3$

Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

b. $h(x) = \sqrt{x-5} + 3$

$h(x) = \sqrt{x-5} + 3$ →

Parent function $f(x) = \sqrt{x}$ →



Transformation:
Translated 3 units up
Translated 5 units right

$D = [5.\infty)$ $R = (3, \infty)$

Sample Problem 3: **Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

c. $h(x) = -|x + 4| - 1$

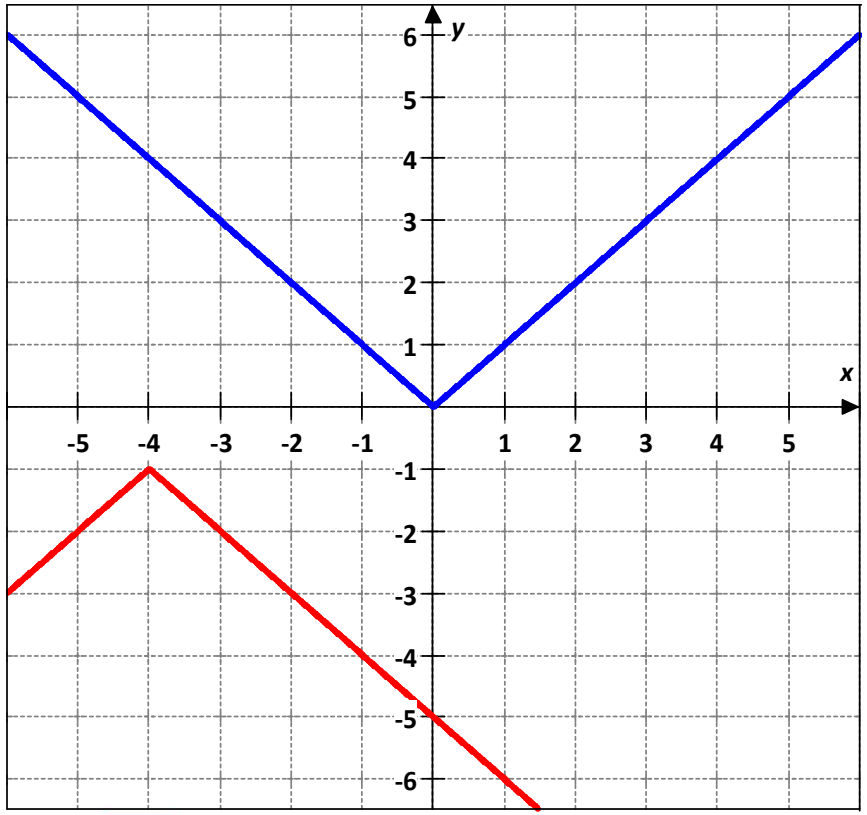
Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

c. $h(x) = -|x + 4| - 1$

$h(x) = -|x + 4| - 1$ ⟶

Parent function $f(x) = |x|$ ⟶

Transformation:
Reflected in the x axis
Translated 1 unit down
Translated 4 units left



$D = (-\infty . \infty)$ $R = (-\infty, -1]$

Transformations with Absolute Value

$$h(x) = |f(x)|$$

This transformation reflects any portion of the graph of $f(x)$ that is below the x -axis so that it is above the x -axis.

Transformations with Absolute Value

$$h(x) = f(|x|)$$

This transformation results, in the portion of the graph of $f(x)$ that is to the left of the y-axis, being replaced by a reflection of the portion to the right of the y -axis.

Sample Problem 4: Graph each function.

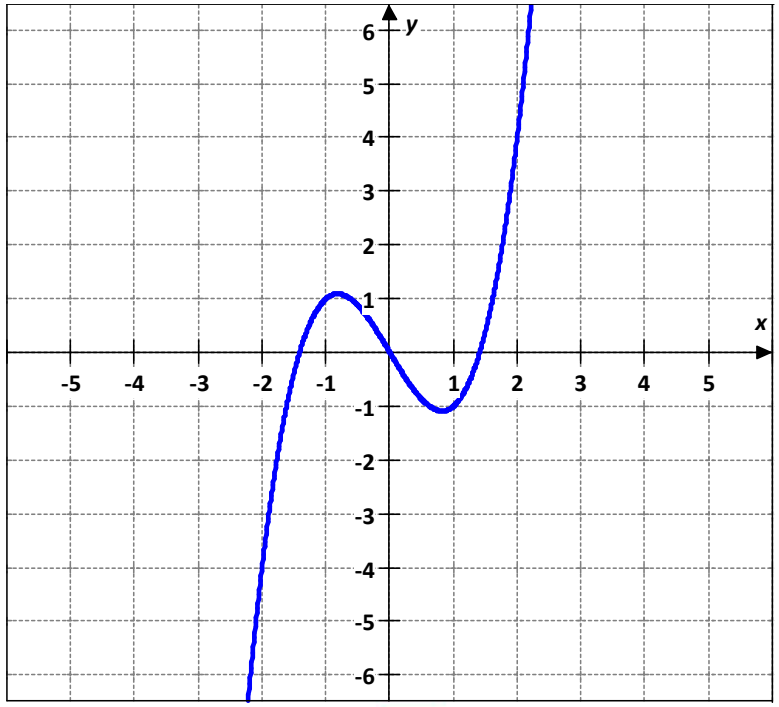
a. $f(x) = x^3 - 2x$ **Graph** $h(x) = |x^3 - 2x|$

Parent Functions and Transformations

Sample Problem 4: Graph each function.

a. $f(x) = x^3 - 2x$ **Graph** $h(x) = |x^3 - 2x|$

$f(x) = x^3 - 2x$ ⟶

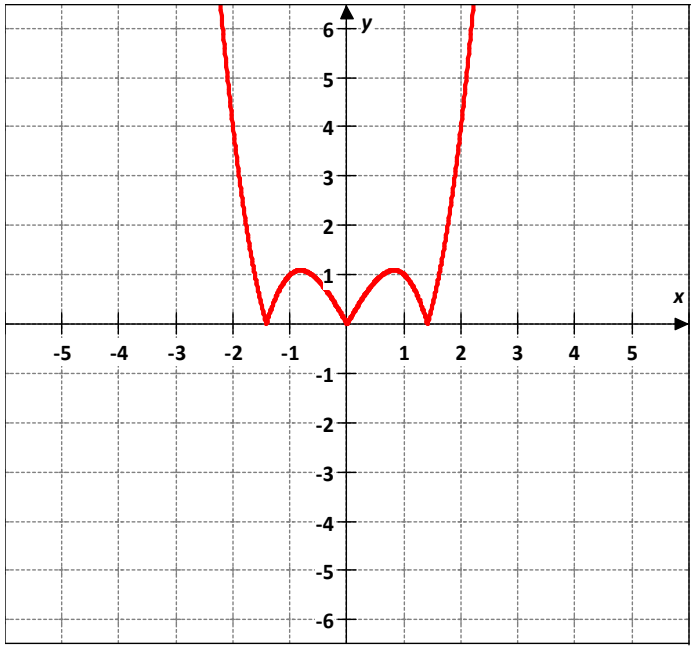


Parent Functions and Transformations

Sample Problem 4: Graph each function.

a. $f(x) = x^3 - 2x$ **Graph** $h(x) = |x^3 - 2x|$

$h(x) = |x^3 - 2x|$ ⟶



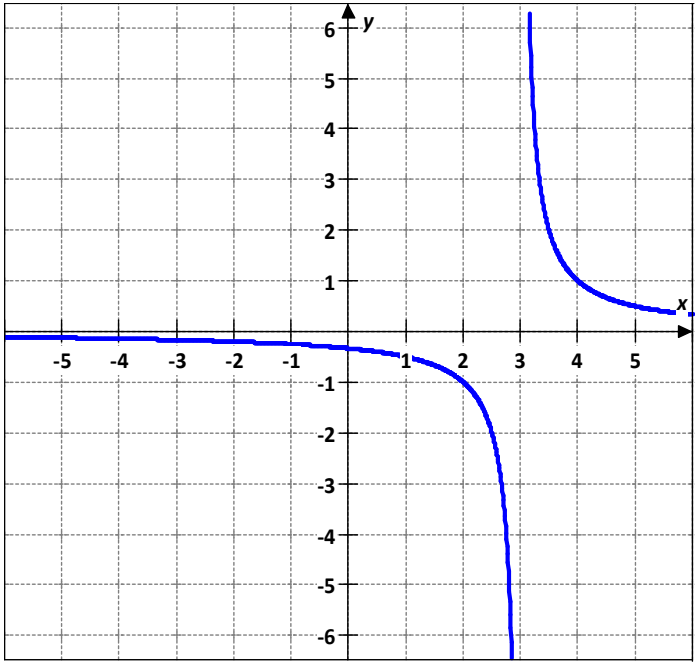
Sample Problem 4: Graph each function.

b. $f(x) = \dfrac{1}{x-3}$ **Graph** $h(x) = \dfrac{1}{|x-3|}$

Sample Problem 4: Graph each function.

b. $f(x) = \dfrac{1}{x-3}$ **Graph** $h(x) = \dfrac{1}{|x-3|}$

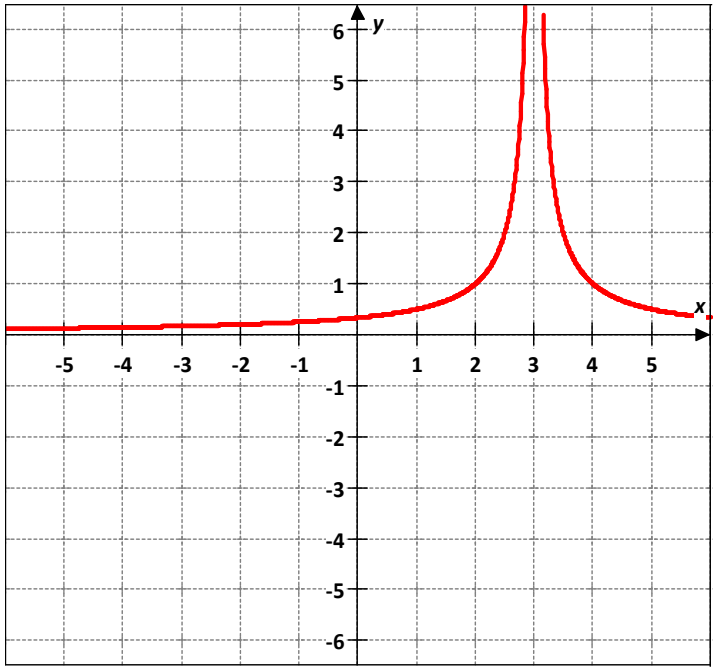
$f(x) = \dfrac{1}{x-3}$ $\longrightarrow$



Sample Problem 4: Graph each function.

b. $f(x) = \dfrac{1}{x-3}$ **Graph** $h(x) = \dfrac{1}{|x-3|}$

$$h(x) = \frac{1}{|x-3|} \longrightarrow$$



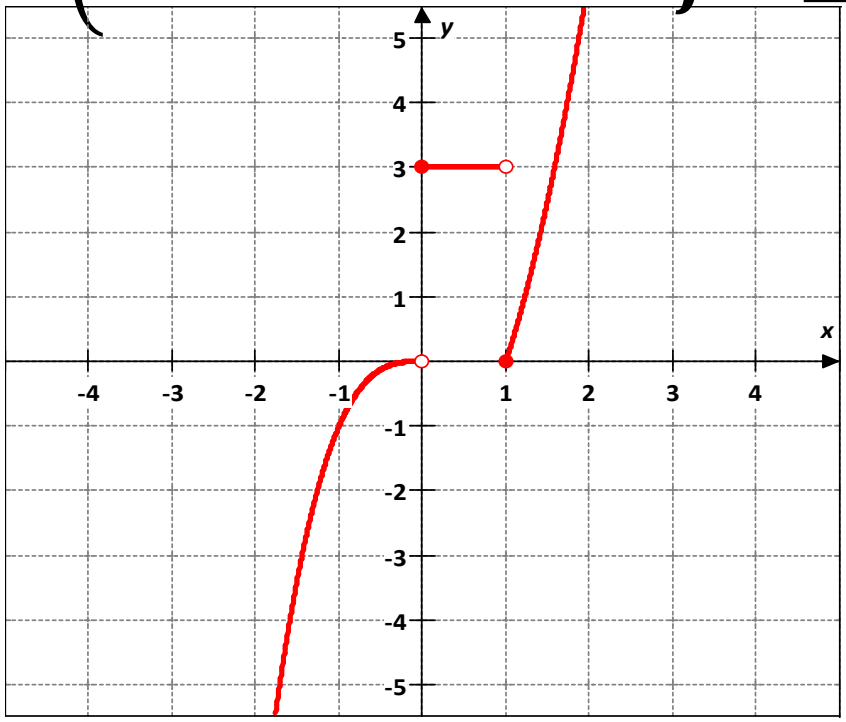
Graph a Piecewise-Defined Function

Sample Problem 5: Graph each piecewise function.

a. $$f(x) = \begin{cases} -x^3 & if\ x < 0 \\ 3 & if\ 0 \le x < 1 \\ 2x^2 - 2 & if\ x \ge 1 \end{cases}$$

Sample Problem 5: Graph each piecewise function.

a. $$f(x) = \begin{cases} -x^3 & if\ x < 0 \\ 3 & if\ 0 \le x < 1 \\ 2x^2 - 2 & if\ x \ge 1 \end{cases}$$



Sample Problem 5: Graph each piecewise function.

b. $$f(x) = \begin{cases} 3x^2 & if\ x \leq -1 \\ -2 & if\ -1 < x < 2 \\ |x^2 - 1| & if\ x \geq 2 \end{cases}$$

Sample Problem 5: Graph each piecewise function.

b. $$f(x) = \begin{cases} 3x^2 & if\ x \le -1 \\ -2 & if\ -1 < x < 2 \\ |x^2 - 1| & if\ x \ge 2 \end{cases}$$