

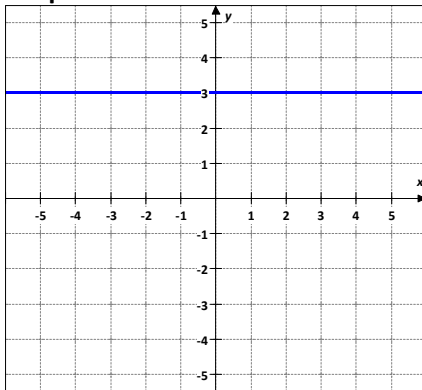
Parent Functions and Transformations Guided Notes

A **family of functions** is a group of functions with graphs that display one or more similar characteristics.

The **Parent Function** is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

Family - Constant Function

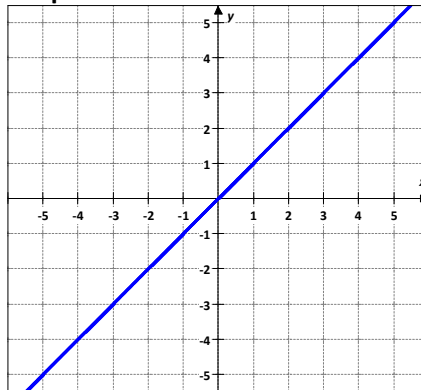
Graph



Rule $f(x) = c$
 Domain = $(-\infty, \infty)$
 Range = $[c]$

Family - Linear Function

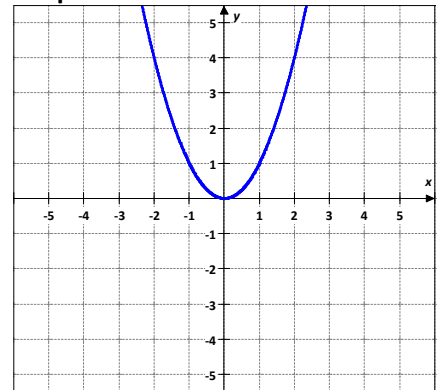
Graph



Rule $f(x) = x$
 Domain = $(-\infty, \infty)$
 Range = $(-\infty, \infty)$

Family - Quadratic Function

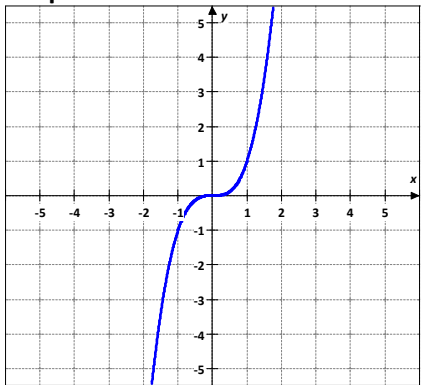
Graph



Rule $f(x) = x^2$
 Domain = $(-\infty, \infty)$
 Range = $[0, \infty)$

Family - Cubic Function

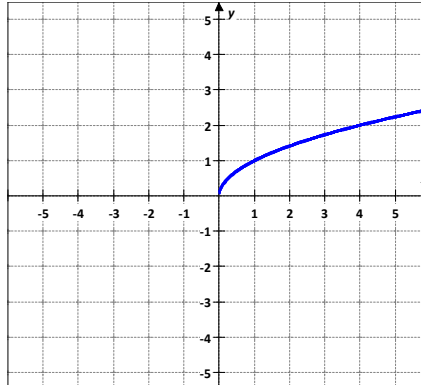
Graph



Rule $f(x) = x^3$
 Domain = $(-\infty, \infty)$
 Range = $(-\infty, \infty)$

Family - Square Root Function

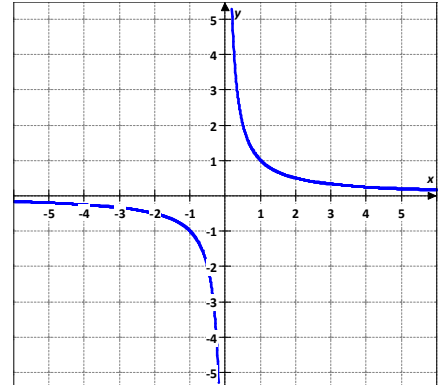
Graph



Rule $f(x) = \sqrt{x}$
 Domain = $[0, \infty)$
 Range = $[0, \infty)$

Family - Reciprocal Function

Graph

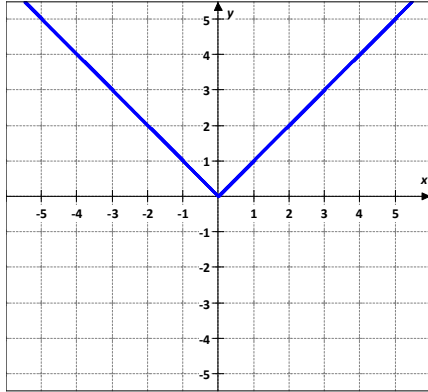


Rule $f(x) = \frac{1}{x}$
 Domain = $(-\infty, 0) \cup (0, \infty)$
 Range = $(-\infty, 0) \cup (0, \infty)$

Parent Functions and Transformations Guided Notes

Family – Absolut Value Function

Graph



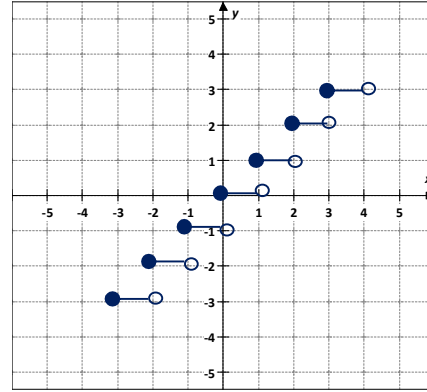
Rule $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

Domain = $(-\infty, \infty)$

Range = $[0, \infty)$

Family - Greatest Integer Function

Graph



Rule $f(x) = \lfloor x \rfloor$

Domain = $(-\infty, \infty)$

Range *All Integer*

Transformations

Transformations

A change in the size or position of a figure or graph of the function is called a transformation.

Rigid transformations change only the position of the graph, leaving the size and shape unchanged.

	Appearance in Function	Transformation of Graph	Transformation of Point
Vertical Translations	$f(x) \rightarrow f(x) + a$ $f(x) \rightarrow f(x) - a$	<i>a units up</i> <i>a units down</i>	$(x, y) \rightarrow (x, y + a)$ $(x, y) \rightarrow (x, y - a)$
Horizontal Translations	$f(x) \rightarrow f(x - b)$ $f(x) \rightarrow f(x + b)$	<i>b units right</i> <i>b units left</i>	$(x, y) \rightarrow (x + b, y)$ $(x, y) \rightarrow (x - b, y)$
Reflections in x-axes	$f(x) \rightarrow -f(x)$	<i>reflected in the x axis</i>	$(x, y) \rightarrow (x, -y)$
Reflections in y-axes	$f(x) \rightarrow f(-x)$	<i>reflected in the y axis</i>	$(x, y) \rightarrow (-x, y)$

Non rigid transformations distort the shape of the graph.

	Appearance in Function	Transformation of Graph	Transformation of Point
Vertical Dilations	$f(x) \rightarrow cf(x) \quad c > 1$ $f(x) \rightarrow cf(x) \quad 0 < c < 1$	<i>expanded vertically</i> <i>compressed vertically</i>	$(x, y) \rightarrow (x, cy)$
Horizontal Dilations	$f(x) \rightarrow f(dx) \quad d > 1$ $f(x) \rightarrow f(dx) \quad 0 < d < 1$	<i>compressed horizontally</i> <i>expanded horizontally</i>	$(x, y) \rightarrow \left(\frac{x}{d}, y\right)$

Parent Functions and Transformations Guided Notes

Sample Problem 1: Identify the parent function and describe the transformations.

- | | | |
|----|--------------------|---|
| a. | $f(x) = (x - 1)^2$ | Parent : $f(x) = x^2$
Transformation: Translation 1 unit right |
| b. | $f(x) = x^3 - 5$ | Parent : $f(x) = x^3$
Transformation: Translation 5 units down |
| c. | $f(x) = - x + 4 $ | Parent : $f(x) = x $
Transformation: Reflection in x-axis
Translation 4 units left |
| d. | $f(x) = 3x^2 + 7$ | Parent : $f(x) = x^2$
Transformation: Expand vertically by a factor of 3
Translation 7 units up |

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

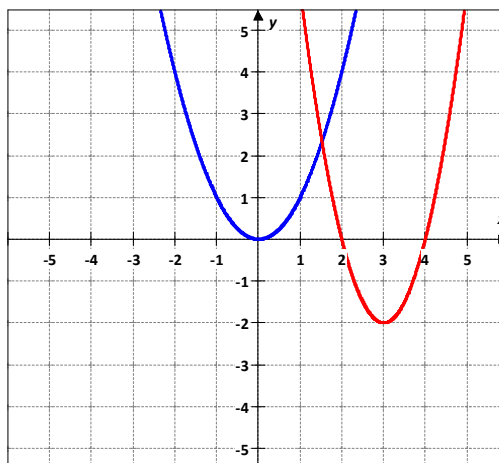
- | | | |
|----|--|-----------------------------|
| a. | Quadratic - expanded horizontally by a factor of 2, translated 7 units up. | $f(x) = \frac{1}{2}x^2 + 7$ |
| b. | Cubic - reflected over the x axis and translated 9 units down. | $f(x) = -x^3 - 9$ |
| c. | Absolute value - translated 3 units up, translated 8 units right. | $f(x) = x - 8 + 3$ |
| d. | Reciprocal - translated 1 unit up. | $f(x) = \frac{1}{x} + 1$ |

Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

- a. $h(x) = 2(x - 3)^2 - 2$
 $h(x) = 2(x - 3)^2 - 2$ →
 Parent function $f(x) = x^2$ →

Transformation:
 Expand vertically by a factor of 2
 Translated 2 units down
 Translated 3 units right

$D = (-\infty, \infty)$
 $R = (-2, \infty)$



Parent Functions and Transformations Guided Notes

b. $h(x) = \sqrt{x-5} + 3$

$h(x) = \sqrt{x-5} + 3$ →

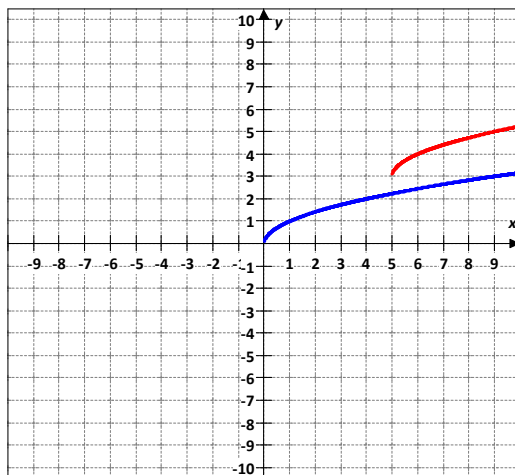
Parent function $f(x) = \sqrt{x}$ →

Transformation:

Translated 3 units up
Translated 5 units right

$D = [5, \infty)$

$R = (3, \infty)$



c. $h(x) = -|x+4| - 1$

$h(x) = -|x+4| - 1$ →

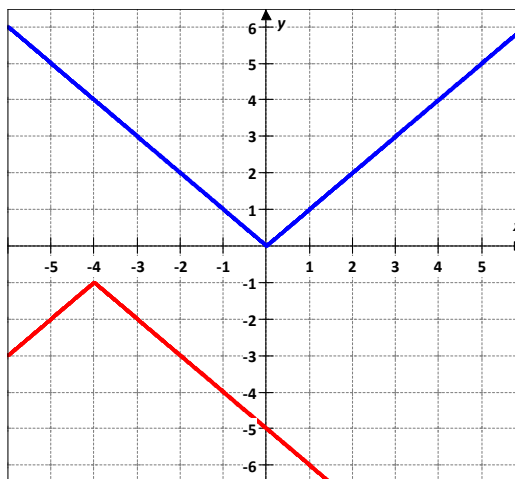
Parent function $f(x) = |x|$ →

Transformation:

Reflected in the x axis
Translated 1 unit down
Translated 4 units left

$D = (-\infty, \infty)$

$R = (-\infty, -1]$



Transformations with Absolute Value

$h(x) = |f(x)|$

This transformation reflects any portion of the graph of $f(x)$ that is below the x -axis so that it is above the x -axis.

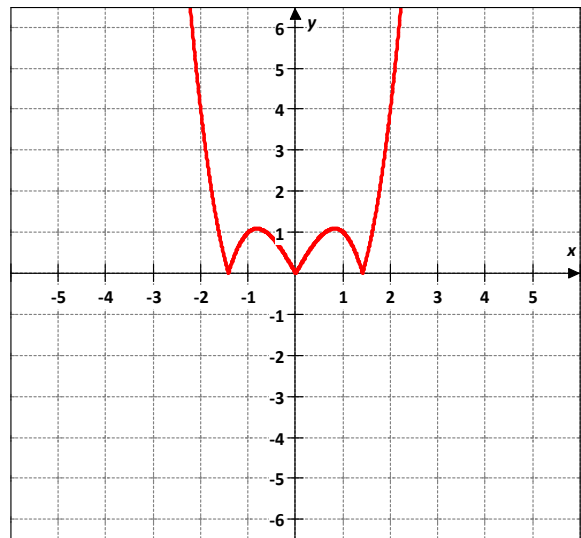
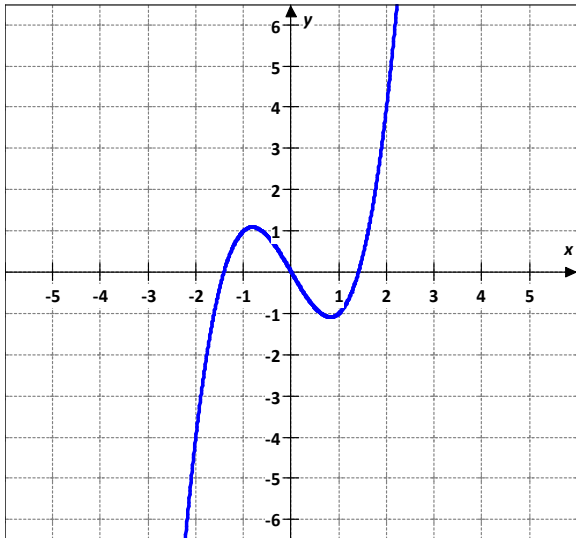
$h(x) = f(|x|)$

This transformation results, in the portion of the graph of $f(x)$ that is to the left of the y -axis, being replaced by a reflection of the portion to the right of the y -axis.

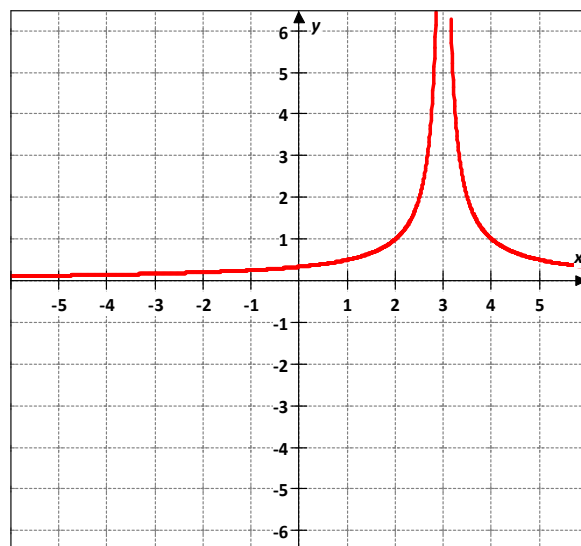
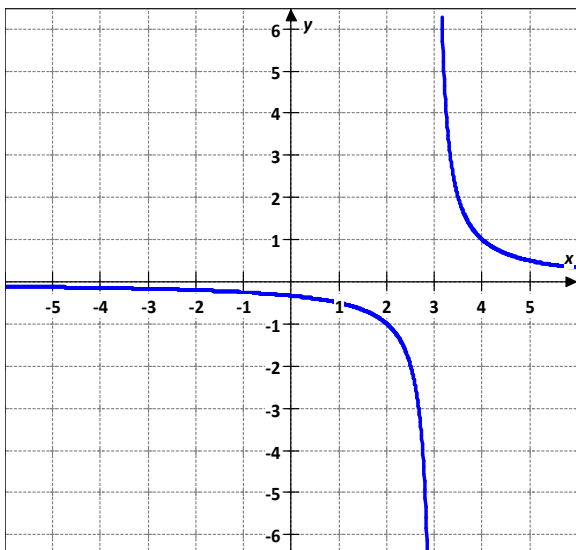
Parent Functions and Transformations Guided Notes

Sample Problem 4: Graph each function.

a. $f(x) = x^3 - 2x$ Graph $h(x) = |x^3 - 2x|$
 $f(x) = x^3 - 2x$ $\xrightarrow{\text{blue}}$
 $h(x) = |x^3 - 2x|$ $\xrightarrow{\text{red}}$



b. $f(x) = \frac{1}{x-3}$ Graph $h(x) = \frac{1}{|x-3|}$
 $f(x) = \frac{1}{x-3}$ $\xrightarrow{\text{blue}}$
 $h(x) = \frac{1}{|x-3|}$ $\xrightarrow{\text{red}}$

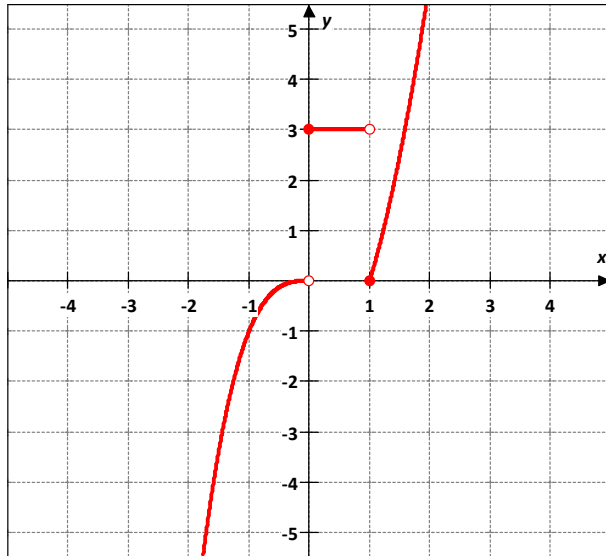


Parent Functions and Transformations Guided Notes

Graph a Piecewise-Defined Function

Sample Problem 5: Graph each piecewise function.

a.
$$f(x) = \begin{cases} -x^3 & \text{if } x < 0 \\ 3 & \text{if } 0 \leq x < 1 \\ 2x^2 - 2 & \text{if } x \geq 1 \end{cases}$$



b.
$$f(x) = \begin{cases} 3x^2 & \text{if } x \leq -1 \\ -2 & \text{if } -1 < x < 2 \\ |x^2 - 1| & \text{if } x \geq 2 \end{cases}$$

