

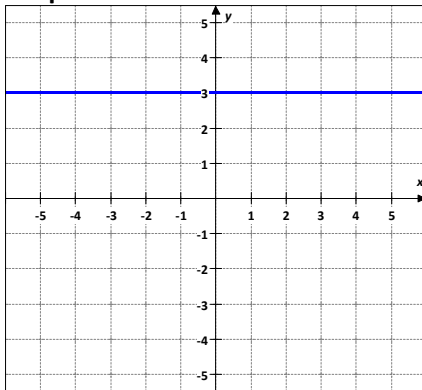
# Parent Functions and Transformations Guided Notes

A **family of functions** is a group of functions with graphs that display one or more similar characteristics.

The **Parent Function** is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

## Family - Constant Function

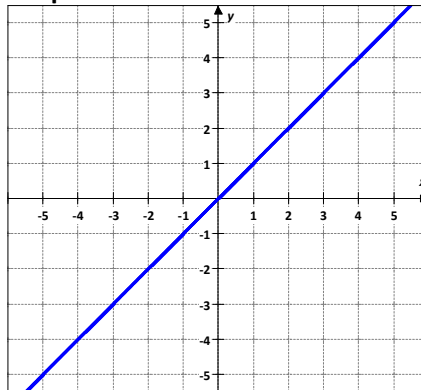
Graph



Rule  $f(x) = c$   
 Domain =  $(-\infty, \infty)$   
 Range =  $[c]$

## Family - Linear Function

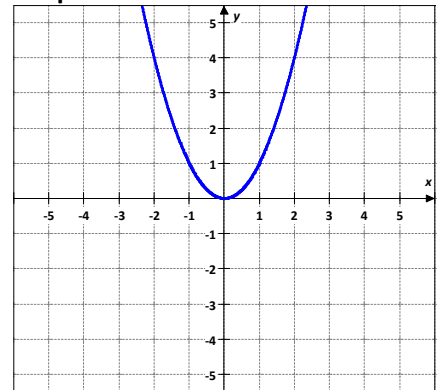
Graph



Rule  $f(x) = x$   
 Domain =  $(-\infty, \infty)$   
 Range =  $(-\infty, \infty)$

## Family - Quadratic Function

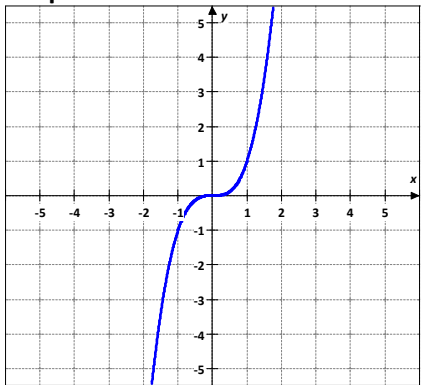
Graph



Rule  $f(x) = x^2$   
 Domain =  $(-\infty, \infty)$   
 Range =  $[0, \infty)$

## Family - Cubic Function

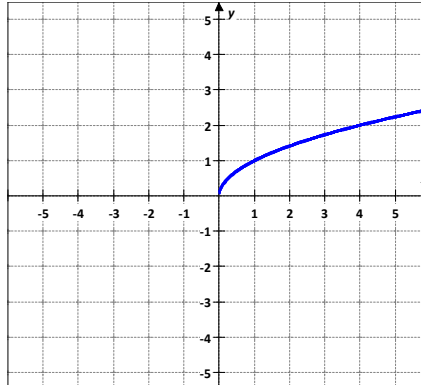
Graph



Rule  $f(x) = x^3$   
 Domain =  $(-\infty, \infty)$   
 Range =  $(-\infty, \infty)$

## Family - Square Root Function

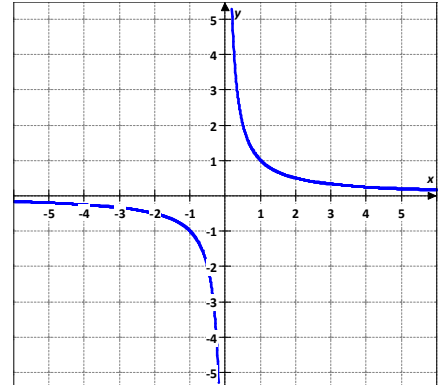
Graph



Rule  $f(x) = \sqrt{x}$   
 Domain =  $[0, \infty)$   
 Range =  $[0, \infty)$

## Family - Reciprocal Function

Graph

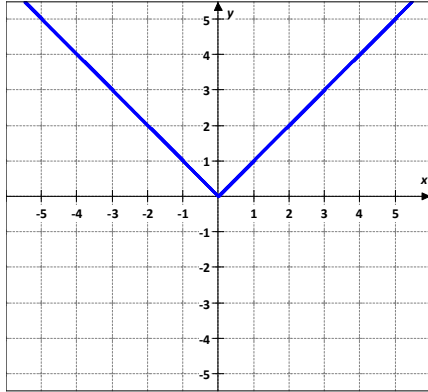


Rule  $f(x) = \frac{1}{x}$   
 Domain =  $(-\infty, 0) \cup (0, \infty)$   
 Range =  $(-\infty, 0) \cup (0, \infty)$

# Parent Functions and Transformations Guided Notes

## Family – Absolut Value Function

Graph



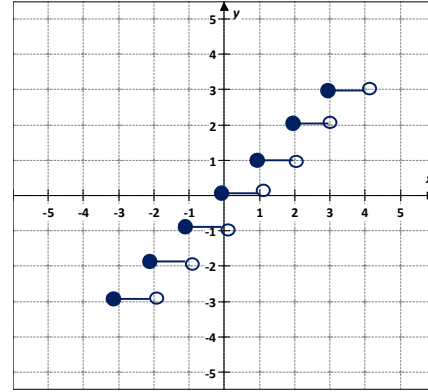
Rule  $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

Domain =  $(-\infty, \infty)$

Range =  $[0, \infty)$

## Family - Greatest Integer Function

Graph



Rule  $f(x) = \lfloor x \rfloor$

Domain =  $(-\infty, \infty)$

Range *All Integer*

## Transformations

### Transformations

A change in the size or position of a figure or graph of the function is called a transformation.

**Rigid transformations** change only the position of the graph, leaving the size and shape unchanged.

	Appearance in Function	Transformation of Graph	Transformation of Point
<b>Vertical Translations</b>	$f(x) \rightarrow f(x) + a$ $f(x) \rightarrow f(x) - a$	<i>a units up</i> <i>a units down</i>	$(x, y) \rightarrow (x, y + a)$ $(x, y) \rightarrow (x, y - a)$
<b>Horizontal Translations</b>	$f(x) \rightarrow f(x - b)$ $f(x) \rightarrow f(x + b)$	<i>b units right</i> <i>b units left</i>	$(x, y) \rightarrow (x + b, y)$ $(x, y) \rightarrow (x - b, y)$
<b>Reflections in x-axes</b>	$f(x) \rightarrow -f(x)$	<i>reflected in the x axis</i>	$(x, y) \rightarrow (x, -y)$
<b>Reflections in y-axes</b>	$f(x) \rightarrow f(-x)$	<i>reflected in the y axis</i>	$(x, y) \rightarrow (-x, y)$

**Non rigid transformations** distort the shape of the graph.

	Appearance in Function	Transformation of Graph	Transformation of Point
<b>Vertical Dilations</b>	$f(x) \rightarrow cf(x) \quad c > 1$ $f(x) \rightarrow cf(x) \quad 0 < c < 1$	<i>expanded vertically</i> <i>compressed vertically</i>	$(x, y) \rightarrow (x, cy)$
<b>Horizontal Dilations</b>	$f(x) \rightarrow f(dx) \quad d > 1$ $f(x) \rightarrow f(dx) \quad 0 < d < 1$	<i>compressed horizontally</i> <i>expanded horizontally</i>	$(x, y) \rightarrow \left(\frac{x}{d}, y\right)$

# Parent Functions and Transformations Guided Notes

**Sample Problem 1:** Identify the parent function and describe the transformations.

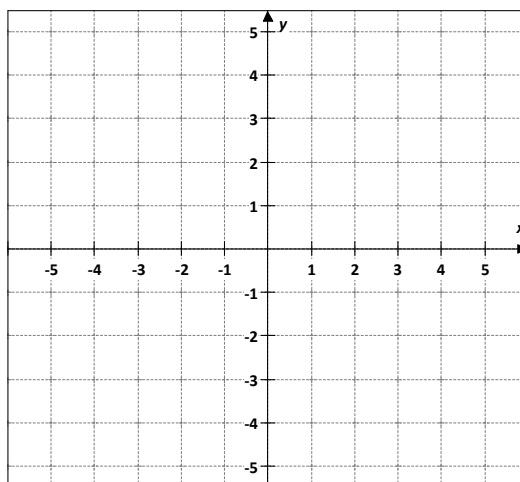
- |    |                    |                             |
|----|--------------------|-----------------------------|
| a. | $f(x) = (x - 1)^2$ | Parent :<br>Transformation: |
| b. | $f(x) = x^3 - 5$   | Parent :<br>Transformation: |
| c. | $f(x) = - x + 4 $  | Parent<br>Transformation:   |
| d. | $f(x) = 3x^2 + 7$  | Parent :<br>Transformation: |

**Sample Problem 2:** Given the parent function and a description of the transformation, write the equation of the transformed function  $f(x)$ .

- a. Quadratic - expanded horizontally by a factor of 2, translated 7 units up.
- b. Cubic - reflected over the x axis and translated 9 units down.
- c. Absolute value - translated 3 units up, translated 8 units' right.
- d. Reciprocal - translated 1 unit up.

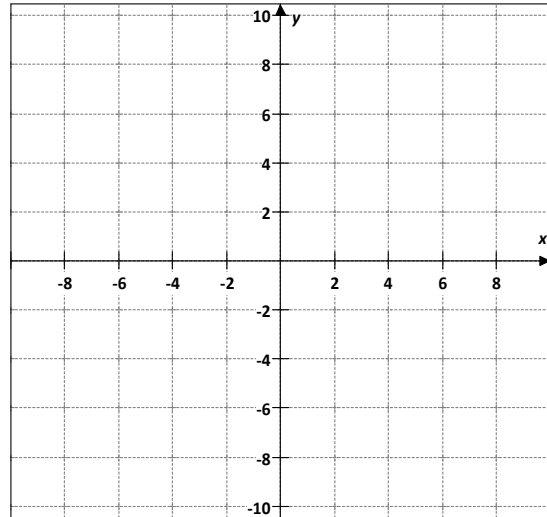
**Sample Problem 3:** Use the graph of parent function to graph each function. Find the domain and the range of the new function.

a.  $h(x) = 2(x - 3)^2 - 2$

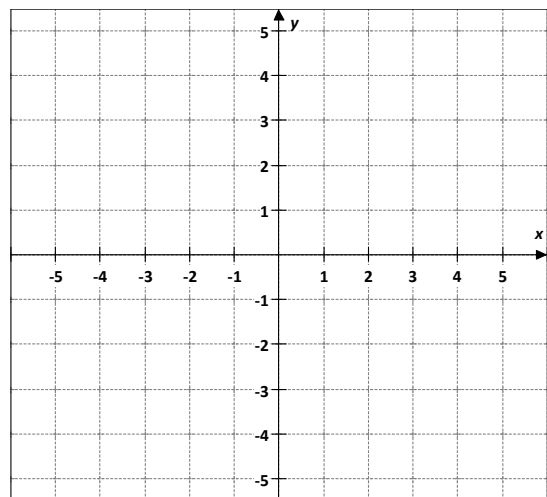


# Parent Functions and Transformations Guided Notes

b.  $h(x) = \sqrt{x - 5} + 3$



c.  $h(x) = -|x + 4| - 1$



## Transformations with Absolute Value

$h(x) = |f(x)|$

This transformation reflects any portion of the graph of  $f(x)$  that is below the  $x$ -axis so that it is above the  $x$ -axis.

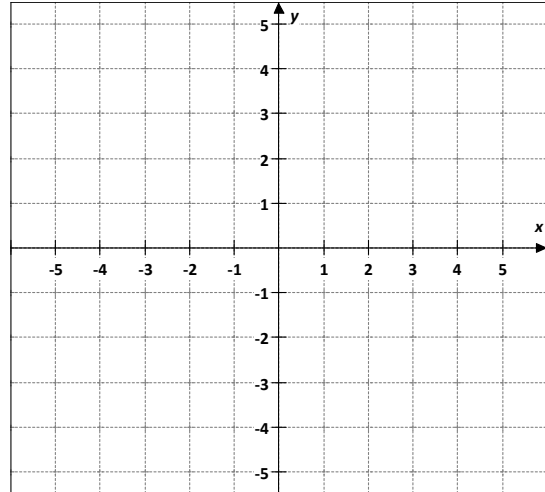
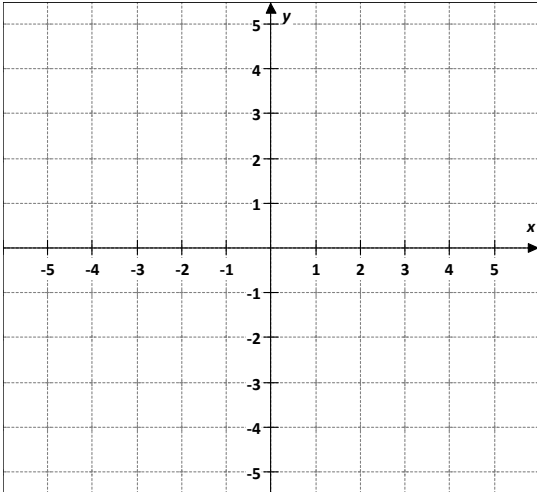
$h(x) = f(|x|)$

This transformation results, in the portion of the graph of  $f(x)$  that is to the left of the  $y$ -axis, being replaced by a reflection of the portion to the right of the  $y$ -axis.

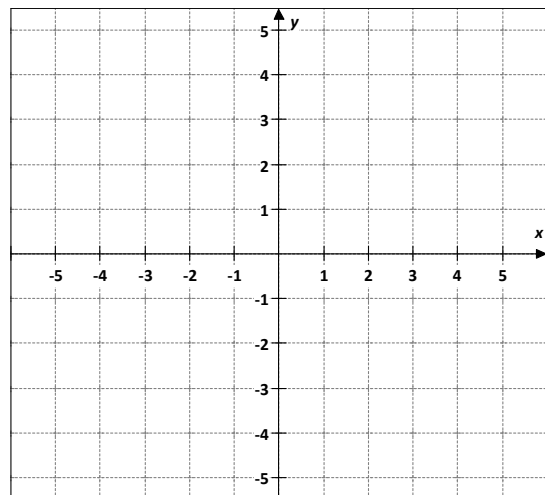
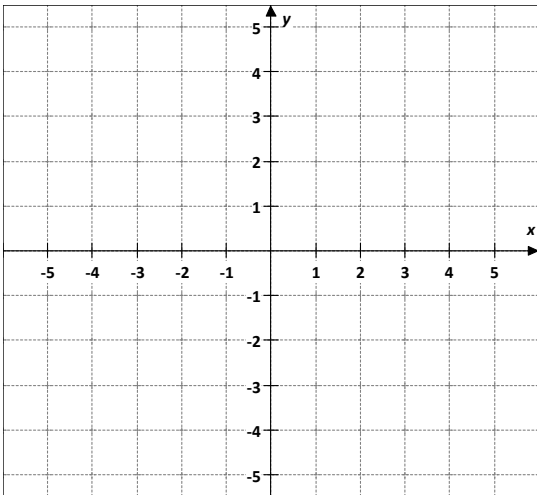
# Parent Functions and Transformations Guided Notes

**Sample Problem 4:** Graph each function.

a.  $f(x) = x^3 - 2x$  Graph  $h(x) = |x^3 - 2x|$



b.  $f(x) = \frac{1}{x-3}$  Graph  $h(x) = \frac{1}{|x-3|}$

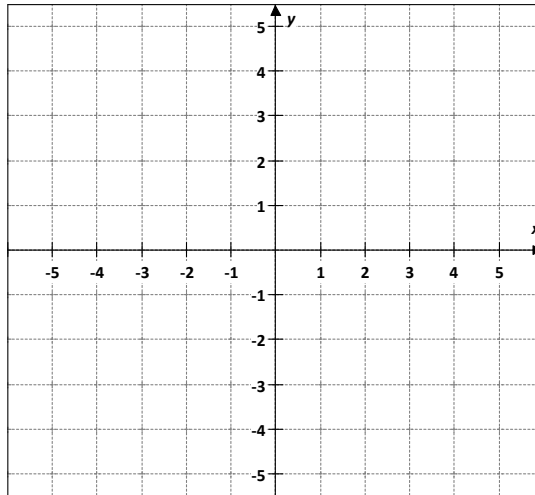


# Parent Functions and Transformations Guided Notes

## Graph a Piecewise-Defined Function

**Sample Problem 5:** Graph each piecewise function.

a. 
$$f(x) = \begin{cases} -x^3 & \text{if } x < 0 \\ 3 & \text{if } 0 \leq x < 1 \\ 2x^2 - 2 & \text{if } x \geq 1 \end{cases}$$



b. 
$$f(x) = \begin{cases} 3x^2 & \text{if } x \leq -1 \\ -2 & \text{if } -1 < x < 2 \\ |x^2 - 1| & \text{if } x \geq 2 \end{cases}$$

