



Arithmetic Sequences and Series

Unit 10 Lesson 2

Arithmetic Sequences and Series

Students will be able to:

Recognize, write, and find the n th terms of arithmetic sequences.

Find n th partial sums of arithmetic sequences.

Key Vocabulary:

Arithmetic sequence

Common difference

Arithmetic Sequences

An arithmetic sequence is an ordered list of terms in which the difference between consecutive terms is constant.

This constant is called the **common difference d** .

If you subtract the first term from the second term for any two consecutive terms of the sequence, you will arrive at the common difference $d = a_n - a_{n-1}$.

Arithmetic Sequences and Series

Sample Problem 1: Decide whether each sequence is arithmetic.

a. 4, 8 12, 16

Arithmetic Sequences and Series

Sample Problem 1: Decide whether each sequence is arithmetic.

a. 4, 8 12, 16

$$8 - 4 = 4$$

$$12 - 8 = 4$$

$$16 - 12 = 4$$

$$d = 4$$

This sequence is arithmetic.

Arithmetic Sequences and Series

Sample Problem 1: Decide whether each sequence is arithmetic.

b. $-8, -1, 1, 8 \dots \dots \dots$

Arithmetic Sequences and Series

Sample Problem 1: Decide whether each sequence is arithmetic.

b. $-8, -1, 1, 8 \dots \dots \dots$

$$-1 - (-8) = -1 + 8 = 7$$

$$1 - (-1) = 1 + 1 = 2$$

$$8 - 1 = 7$$

This sequence is not arithmetic.

Arithmetic Sequences and Series

The explicit formula for the general term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$.

a_1 – *the first term*

d – *the common difference*

n – *the number of term*

a_n – *the general term or n th term*

The recursive formula for the general term of an arithmetic sequence is $a_n = a_{n-1} + d$.

The terms between any two nonconsecutive terms of an arithmetic sequence are called **arithmetic means**.

Arithmetic Sequences and Series

Sample Problem 2: Find the first four terms and common difference of each arithmetic sequence.

a. $a_n = 2n + 2$

Arithmetic Sequences and Series

Sample Problem 2: Find the first four terms and common difference of each arithmetic sequence.

a. $a_n = 2n + 2$

$$a_1 = 2 * 1 + 2 = 2 + 2 = 4$$

$$a_2 = 2 * 2 + 2 = 4 + 2 = 6$$

$$a_3 = 2 * 3 + 2 = 6 + 2 = 8$$

$$a_4 = 2 * 4 + 2 = 8 + 2 = 10$$

$$d = 12 - 10 = 2$$

$$d = 2$$

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Sample Problem 2: Find the first four terms and common difference of each arithmetic sequence.

b. $a_n = 3 - 2n$

Arithmetic Sequences and Series

Sample Problem 2: Find the first four terms and common difference of each arithmetic sequence.

b. $a_n = 3 - 2n$

$$a_1 = 3 - 2 * 1 = 3 - 2 = 1$$

$$a_2 = 3 - 2 * 2 = 3 - 4 = -1$$

$$a_3 = 3 - 2 * 3 = 3 - 6 = -3$$

$$a_4 = 3 - 2 * 4 = 3 - 8 = -5$$

$$d = 7 - (-5) = -7 + 5 = -2$$

$$d = -2$$

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Sample Problem 3: Find the specified term of each arithmetic sequence.

a. *22th term* =? $a_1 = -1,$ $d = 3$

Arithmetic Sequences and Series

Sample Problem 3: Find the specified term of each arithmetic sequence.

a. ***22th term =?*** $a_1 = -1$ $d = 3$ $n = 22$

$$a_n = a_1 + (n - 1)d$$

$$a_{22} = -1 + (22 - 1) * 3$$

$$a_{22} = -1 + 21 * 3$$

$$a_{22} = -1 + 63$$

$$a_{22} = 62$$

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Sample Problem 3: Find the specified term of each arithmetic sequence.

b. *43th term* =? $a_1 = 13,$ $d = 12$

Arithmetic Sequences and Series

Sample Problem 3: Find the specified term of each arithmetic sequence.

b. *43th term* =? $a_1 = 13$ $d = 12$ $n = 43$

$$a_n = a_1 + (n - 1)d$$

$$a_{43} = 13 + (43 - 1) * 12$$

$$a_{43} = 13 + 42 * 12$$

$$a_{43} = 13 + 504$$

$$a_{43} = 517$$

An Arithmetic Series

An arithmetic series is the indicated sum of the terms of an arithmetic sequence.

$$a_1 + a_2 + a_3 + \dots \dots \dots + a_n$$

The sum of the first n terms of the arithmetic series is called the **n th partial sum** and is denoted S_n .

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} (2a_1 + (n - 1) * d)$$

Arithmetic Sequences and Series

Sample Problem 4: Find the indicated sum for each sequence.

a. $a_1 = 4$, $d = -4$ $S_{12} = ?$

Arithmetic Sequences and Series

Sample Problem 4: Find the indicated sum for each sequence.

a. $a_1 = 4, \quad d = -4, \quad n = 12 \quad S_{12} = ?$

$$S_n = \frac{n}{2} (2a_1 + (n - 1) * d)$$

$$S_{12} = \frac{12}{2} (2 * 4 + (12 - 1) * (-4))$$

$$S_{12} = \frac{12}{2} (8 + 11 * (-4))$$

$$S_{12} = \frac{12}{2} (8 - 44)$$

$$S_{12} = 6 * (-36) \quad S_{12} = -216$$

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Sample Problem 4: Find the indicated sum for each sequence.

b. $-1, 5, 11, 17 \dots \dots \dots S_{30} = ?$

Arithmetic Sequences and Series

Sample Problem 4: Find the indicated sum for each sequence.

$$\text{b. } a_1 = -1, \quad n = 30 \quad S_{30} = ?$$

$$d = 11 - 5 = 6$$

$$S_n = \frac{n}{2} (2a_1 + (n - 1) * d)$$

$$S_{30} = \frac{30}{2} (2 * (-1) + (30 - 1) * 6)$$

$$S_{30} = \frac{30}{2} (-2 + 29 * 6)$$

$$S_{30} = \frac{30}{2} (-2 + 174)$$

$$S_{30} = 15 * 172 \quad S_{30} = 2,580$$