PreCalculusCoach.com

Polar Coordinates

Unit 9 Lesson 1

POLAR COORDINATES

Students will be able to:

Understand the polar coordinates and distance formula for polar coordinates.

Key Vocabulary:

- Polar Coordinates
- Distance Formula for Polar Coordinates

Polar Coordinates

Polar Coordinates are a pair of coordinates locating the position of point in a plane, with the first coordinate being the length of the straight line (r) connecting to the point from the origin and second the angle (θ) made by this line with a fixed line.

Mathematically:

Polar coordinates are represented as $P(r, \theta)$.

Re-writing same Polar Coordinates

Polar coordinates can be re-written by adding or subtracting a certain angle from the given angle. Depending on the angle, the sign with radius changes between positive and negative.

Mathematically,

If there is a polar coordinate $P(r, \theta)$, then similar coordinates can be written by adding(or subtracting) $k\pi(k180°)$ to the given angle.

- If $\boldsymbol{k}$ is even, then the sign of $\boldsymbol{r}$ remains positive.
- If $\boldsymbol{k}$ is odd, then the sign of $\boldsymbol{r}$ becomes negative.

Problem 1: Find a different pair of polar coordinates for the point $(5, 960°)$ such that $0 \leq \theta \leq 180°$ or $0 \leq \theta \leq \pi$.

Problem 1: Find a different pair of polar coordinates for the point $(5, 960°)$ such that $0 \leq \theta \leq 180°$ or $0 \leq \theta \leq \pi$.

Let $P(r, \theta) = P(5,960°)$. We subtract multiples of $180°$ to make the angle between $0°$ and $180°$.

$$960° - 5(180°) = 960° - 900° = 60°$$

Now, $60°$ is between $0°$ and $180°$, also since $k = 5$ is odd, so $r = 5$ is replaced with $r = -5$.

$$\rightarrow \boldsymbol{P(5, 960°) = P(-5, 60°)}$$

Distance Formula for Polar Coordinates

If we have two polar coordinates $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$, the distance between the two points (represented as P_1P_2) is given by:

$$\text{Distance } \boldsymbol{P_1P_2} = \sqrt{\boldsymbol{r_1}^2 + \boldsymbol{r_2}^2 - 2\boldsymbol{r_1 r_2 cos(\theta_2 - \theta_1)}}$$

Problem 2: Find the distance between the points $(2, 30°)$ and $(5, 120°)$.

Problem 2: Find the distance between the points $(2, 30°)$ and $(5, 120°)$.

Let $P_1(r_1, \theta_1) = P_1(2,30°)$ and $P_2(5,120°)$, then:

$$P_1P_2 = \sqrt{2^2 + 5^2 - 2(2)(5)\cos(120° - 30°)}$$

$$P_1P_2 = \sqrt{29 - 20\cos(90°)}$$

$$P_1P_2 = \sqrt{29 - 20(0)} = \sqrt{29}$$

$$\rightarrow \boldsymbol{P_1P_2 = 5.39}$$