Introduction to Vectors

Unit 8 Lesson 1

Students will be able to:

Find equal, opposite, and parallel vectors.

Add and subtract vectors geometrically.

Multiply a vector by a scalar.

Key Vocabulary:

Equal vectors

Opposite vectors

Parallel vectors

A vector is a quantity that has both magnitude and direction.

The magnitude of a vector is the length of a directed line segment, and the direction of a vector is the directed angle between the positive x-axis and the vector.

A scalar is a quantity that describes magnitude or size only (with or without units). It does not include direction.

A vector which has a magnitude of 1 is called a **unit vector**.

Sample Problem 1: **State whether each quantity described is a vector quantity or a scalar quantity.**

a. a weight of $50\ N$.

Sample Problem 1: **State whether each quantity described is a vector quantity or a scalar quantity.**

a.   a weight of **50 N**.

This quantity has a magnitude of **50 N**, but no direction is given.
This is a **scalar quantity.**

Sample Problem 1: **State whether each quantity described is a vector quantity or a scalar quantity.**

b.   A rabbit running **20** meters per second due east.

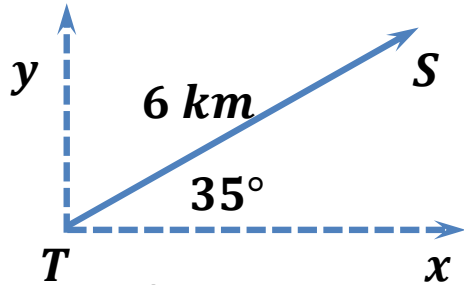**Sample Problem 1:** **State whether each quantity described is a vector quantity or a scalar quantity.**

b. A rabbit running **20** meters per second due east.

This quantity has a magnitude of **20** meters per second and a direction of due east.
This is a **vector quantity.**

f(x) PreCalculusCoach.com

A vector can be represented in several ways:

- In words, for example, as $6\ km$ at an angle of $35°$ to the horizontal.

- In a diagram, as a geometric vector, this is a representation of a vector using an arrow diagram.

- In symbols, using the endpoints of the arrow $\overrightarrow{TS}$, or using a single letter $\vec{v}$.

The magnitude, or size, of a vector is designated using absolute value brackets. The magnitude of vector $\overrightarrow{TS}$ or $\vec{v}$ is written as $\left|\overrightarrow{TS}\right|$ or $|\vec{v}|$.

A vector's direction can be expressed in the diagram of $\overrightarrow{TS}$, it is expressed as an angle, moving counterclockwise with respect to a horizontal line. In navigation, vector directions are expressed as bearings.
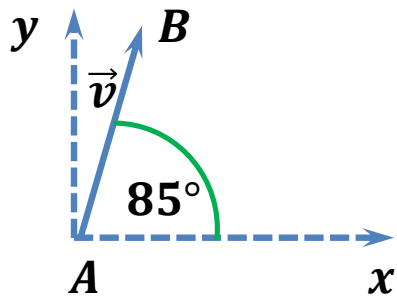
Sample Problem 2: **Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.**

a. $v = 60$ **pounds of force at $85°$ to the horizontal.**

Sample Problem 2: **Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.**

a. $v = 60$ **pounds of force at $85°$ to the horizontal.**

Scale $1cm$: 20 *pounds*

Sample Problem 2: **Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.**

b. $u = 30 \, meters \, at \, a \, bearing \, of \, S \, 45°E$

Sample Problem 2: **Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.**

b. $u = 30 \ meters \ at \ a \ bearing \ of \ S \ 45°E$

N

W   E

45° $\vec{u}$

$Scale \ 1 \ cm: 10 \ m$

S
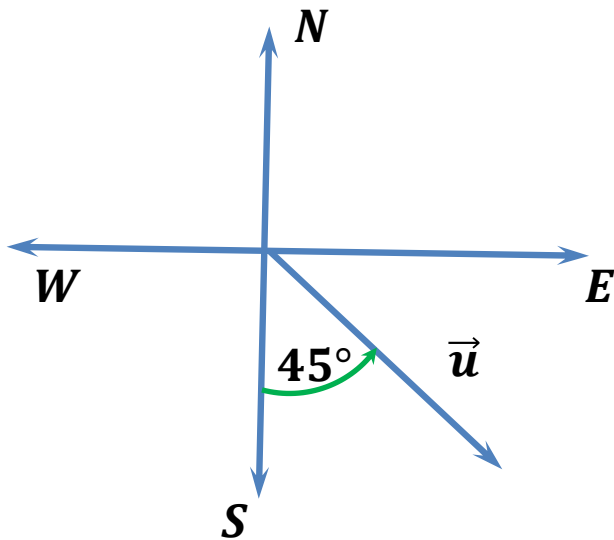
Parallel vectors have the same or opposite direction, but not necessarily the same magnitude.

Equivalent vectors have the same magnitude and the same direction. The location of the vectors does not matter.

Opposite vectors have the same magnitude but opposite direction. The location of the vectors does not matter.

**Sample Problem 3:** **Name the entire equivalent, parallel and opposite vectors in each diagram.**

a.

Sample Problem 3: **Name the entire equivalent, parallel and opposite vectors in each diagram.**

a.

Equivalent vectors

$$\overrightarrow{MN} \text{ and } \overrightarrow{AB}$$
$$\overrightarrow{NB} \text{ and } \overrightarrow{MA}$$

Parallel vectors

$$\overrightarrow{MN} \text{ and } \overrightarrow{AB}$$
$$\overrightarrow{NB} \text{ and } \overrightarrow{MA}$$

No opposite vectors

Sample Problem 3: **Name the entire equivalent, parallel and opposite vectors in each diagram.**

b.

Sample Problem 3: Name the entire equivalent, parallel and opposite vectors in each diagram.

b.
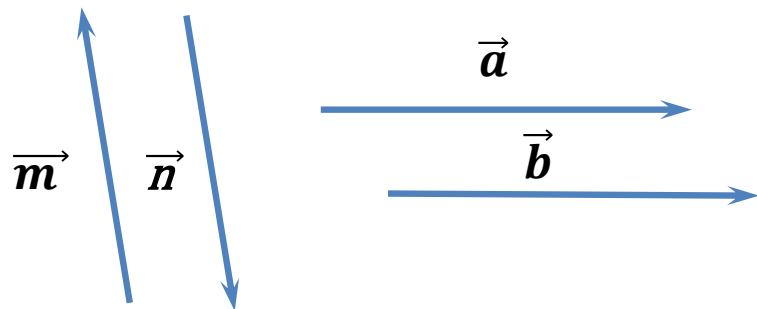
Equivalent vectors

$$\vec{a} \ and \ \vec{b}$$

Parallel vectors

$$\vec{a} \ and \ \vec{b} \qquad \vec{n} \ and \ \overrightarrow{m}$$

Opposite vectors

$$\vec{n} \ and \ \overrightarrow{m}$$

PreCalculusCoach.com

Addition of Vectors

The sum of two or more vectors is called the **resultant** of the vectors.

The resultant can be found using either **the parallelogram method** or **the triangle method**.

Addition of Vectors

Parallelogram Method
Draw the vectors so that their initial points coincide. Then draw lines to form a complete parallelogram. The diagonal from the initial point to the opposite vertex of the parallelogram is the resultant.

$\vec{n}$

$\vec{m} + \vec{n}$

$\vec{m}$
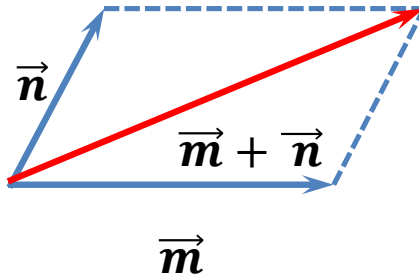
Addition of Vectors

Triangle Method
Draw the vectors one after another, placing the initial point of each successive vector at the terminal point of the previous vector. Then draw the resultant from the initial point of the first vector to the terminal point of the last vector. This method is also called the   Tip-to-tail method.

$$\overrightarrow{m} + \overrightarrow{n} \qquad \overrightarrow{n}$$

$$\overrightarrow{m}$$

Sample Problem 4: **Find the resultant of each pair of vectors using either the triangle or parallelogram method.**

a. $\vec{b} + \vec{a} = ?$

$\vec{a}$

$\vec{b}$

Sample Problem 4: **Find the resultant of each pair of vectors using either the triangle or parallelogram method.**

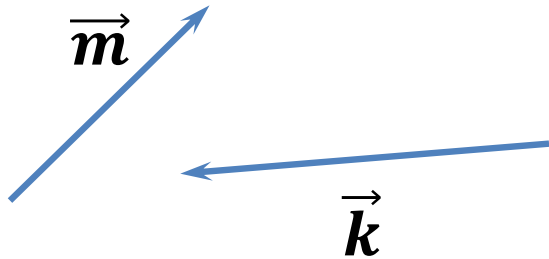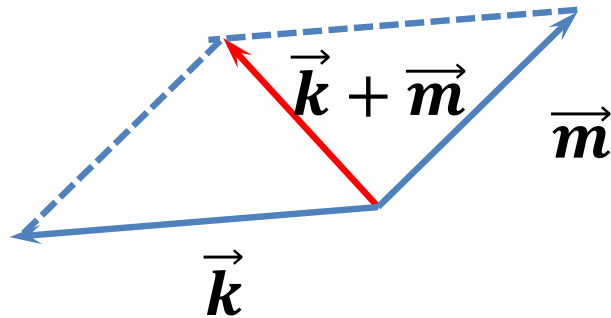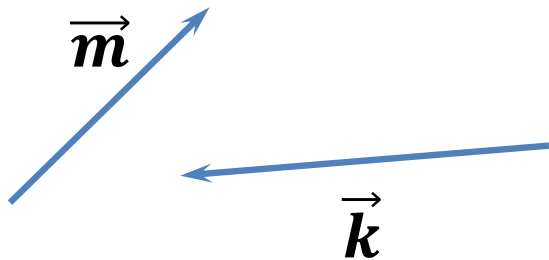a. $\quad \vec{b} + \vec{a} = ?$

Sample Problem 4: **Find the resultant of each pair of vectors using either the triangle or parallelogram method.**
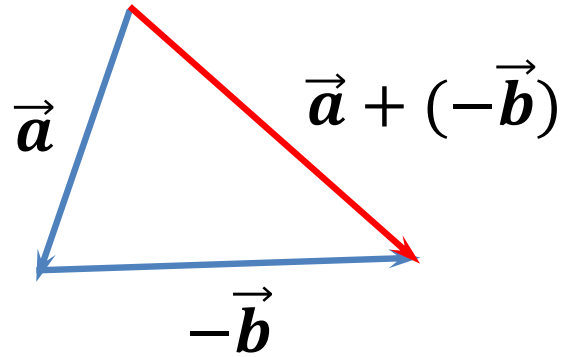
b. $\vec{k} + \vec{m} = ?$

Sample Problem 4: **Find the resultant of each pair of vectors using either the triangle or parallelogram method.**

b. $\quad \vec{k} + \vec{m} = ?$

$\vec{m}$

$\vec{k}$

$\vec{k} + \vec{m}$

$\vec{m}$

$\vec{k}$
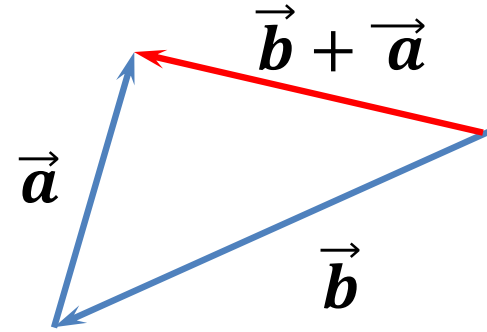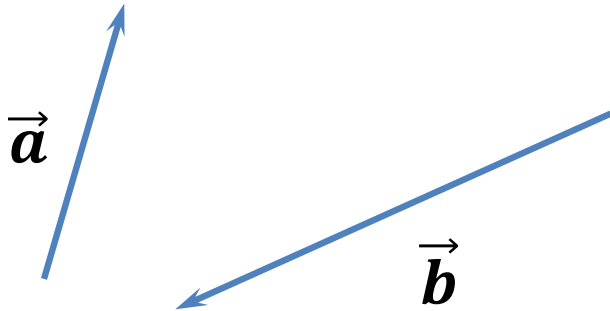
f(x) **PreCalculusCoach.com**

Subtraction of Vectors

The relationship between addition and subtraction with vectors is similar to the relationship between addition and subtraction with scalars.

$$\vec{a} - \vec{b} = \vec{a} + \left(-\vec{b}\right)$$

Sample Problem 5: Find the resultant of each pair of vectors using either the triangle or parallelogram method.
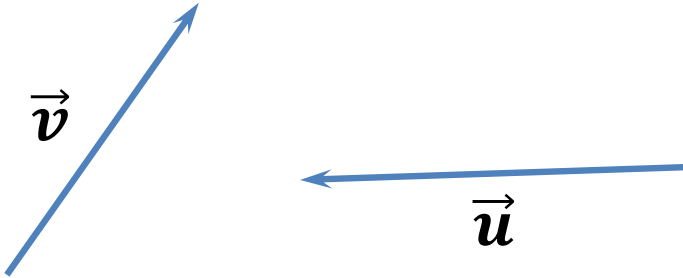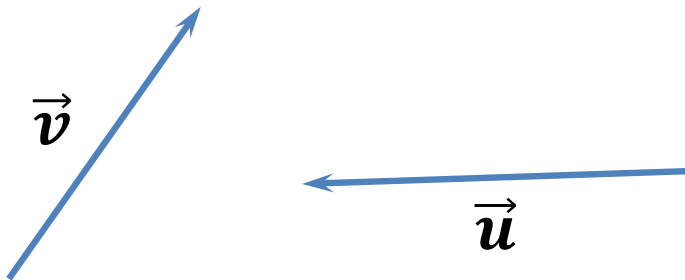
a. $\vec{a} - \vec{b} = ?$

Sample Problem 5: Find the resultant of each pair of vectors using either the triangle or parallelogram method.

a. $\quad \vec{a} - \vec{b} = ?$

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

$\vec{a}$

$\vec{b}$

$\vec{a} + (-\vec{b})$

$\vec{a}$

$-\vec{b}$

f(x) **PreCalculusCoach.com**

Sample Problem 5: Find the resultant of each pair of vectors using either the triangle or parallelogram method.

b. $\vec{v} - \vec{u} = ?$

Sample Problem 5: **Find the resultant of each pair of vectors using either the triangle or parallelogram method.**

b. $\quad \vec{v} - \vec{u} = ?$

$$\vec{v} - \vec{u} = \vec{v} + (-\vec{u})$$

$\vec{v}$

$\vec{u}$

$-\vec{u}$

$\vec{v}$

$\vec{v} + (-\vec{u})$
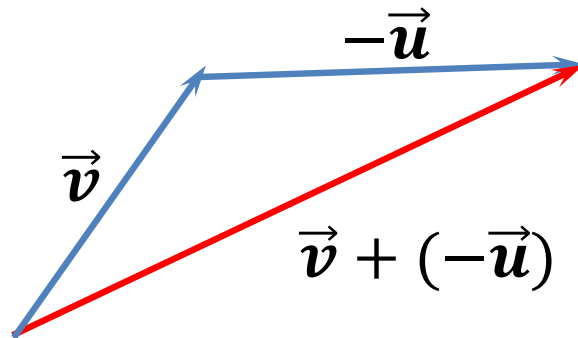
Multiplying a Vector by a Scalar

If k is any positive scalar and $\vec{a}$ is a vector then $k\vec{a}$ is a vector in the same direction as a but k times longer. If k is negative, $k\vec{a}$ is a vector in the opposite direction to a and k times longer.
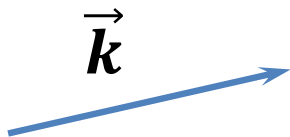
PreCalculusCoach.com

Sample Problem 6: **Multiply each vector.**

a. $2\vec{k} =?$

$\vec{k}$

Sample Problem 6: **Multiply each vector.**

a. $2\vec{k} =?$

$\vec{k}$

$2\vec{k}$

Sample Problem 6: **Multiply each vector.**

b. $-3\vec{s} = ?$
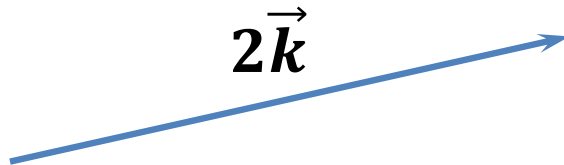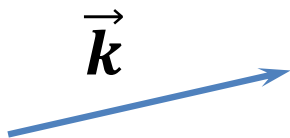
$\vec{s}$
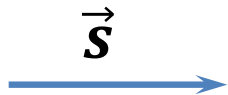
Sample Problem 6: **Multiply each vector.**

b. $-3\vec{s} = ?$

$\vec{s}$
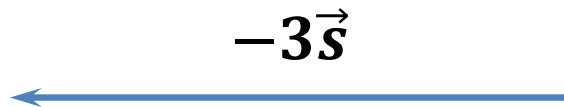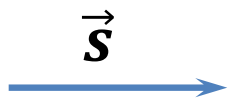
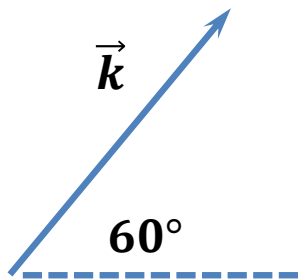$-3\vec{s}$

Two or more vectors whose sum is a given vector are called **components** of the given vector. Components can have any direction. Often it is useful to express a vector as the sum of two perpendicular components.

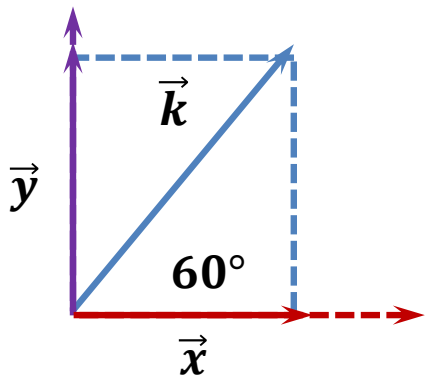**Sample Problem 7:** **Find the magnitude of the horizontal and vertical components of each vector.**

a. $\left|\vec{k}\right| = 10$

Sample Problem 7: **Find the magnitude of the horizontal and vertical components of each vector.**

a. $\left|\vec{k}\right| = 10$

$$\cos 60° = \frac{|\vec{x}|}{|\vec{k}|} \qquad\qquad \sin 60° = \frac{|\vec{y}|}{|\vec{k}|}$$

$$|\vec{x}| = |\vec{k}| * \cos 60° \qquad |\vec{y}| = |\vec{k}| * \sin 60°$$

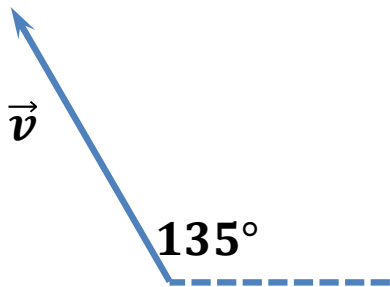$$|\vec{x}| = 10 * \frac{1}{2} \qquad\qquad |\vec{y}| = 10 * \frac{\sqrt{3}}{2}$$

$$|\vec{x}| = 5 \qquad\qquad\qquad |\vec{y}| = 5\sqrt{3}$$

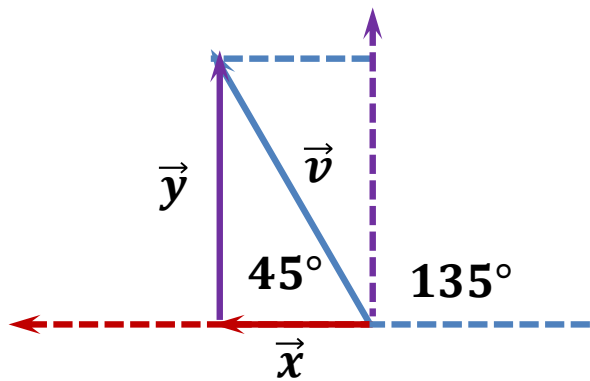**Sample Problem 7:** **Find the magnitude of the horizontal and vertical components of each vector.**

b. $|\vec{v}| = 12$

$\vec{v}$

135°

Sample Problem 7: **Find the magnitude of the horizontal and vertical components of each vector.**

b. $|\vec{v}| = 12$

$$cos\ 45° = \frac{|\vec{x}|}{|\vec{v}|}$$

$$sin\ 45° = \frac{|\vec{y}|}{|\vec{v}|}$$

$$|\vec{x}| = |\vec{v}| * cos\ 45°$$

$$|\vec{y}| = |v| * sin\ 45°$$

$$|\vec{x}| = 12 * \frac{\sqrt{2}}{2}$$

$$|\vec{y}| = 12 * \frac{\sqrt{2}}{2}$$

$$|\vec{x}| = 6\sqrt{2}$$

$$|\vec{y}| = 6\sqrt{2}$$

PreCalculusCoach.com