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Ellipses and Circles

Unit 7 Lesson 2

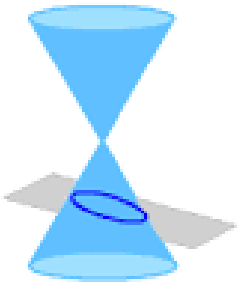
Students will be able to:

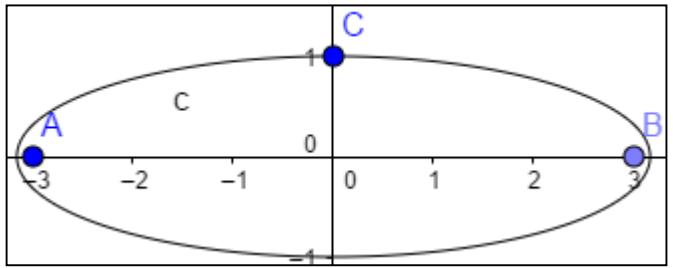
Understand the conics ellipse and circle and their associated properties.

Key Vocabulary:

- Ellipse
- Major Axis, Minor Axis
- Circle
- Radius, Diameter

An **Ellipse** is a curve formed by the intersection of a plane and a double cone such that the plane cuts the cone at an angle.





Ellipse

Equations Representing Ellipses

The equations representing the ellipses are given below:

- **Ellipse with Horizontal Axis ($a > b$)**

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

- **Ellipse with Vertical Axis ($a > b$)**

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Foci of Ellipse

Foci are two fixed points such that the sum of distances between any point on Ellipse and these two points is a constant.

For **vertical ellipse**, foci are given as:

$$(h, k \pm c)$$

For **horizontal ellipse**, foci are given as:

$$(h \pm c, k)$$

where,

$$c^2 = a^2 - b^2$$

Major Axis and Minor Axis

Vertices are the points on the ellipse where the line passing through the foci and called **Major Axis** intersects the ellipse.

Minor Axis is the line perpendicular to major axis and intersects the ellipse at points called **Co-vertices.**

Ellipse	Vertices	Co-Vertices
Vertical	$(h, k \pm a)$	$(h \pm b, k)$
Horizontal	$(h \pm a, k)$	$(h, k \pm b)$

Problem 1: Graph $4x^2 + 25y^2 = 100$. Identify the center, vertices, co-vertices and foci of the ellipse.

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Re-write the equation in standard form:

$$4x^2 + 25y^2 = 100 \rightarrow \frac{x^2}{25} + \frac{y^2}{4} = 1$$

Here, $a = 5$; $b = 2$; $h = 0$; $k = 0$

Ellipse is horizontal, since a is with $x - term$.

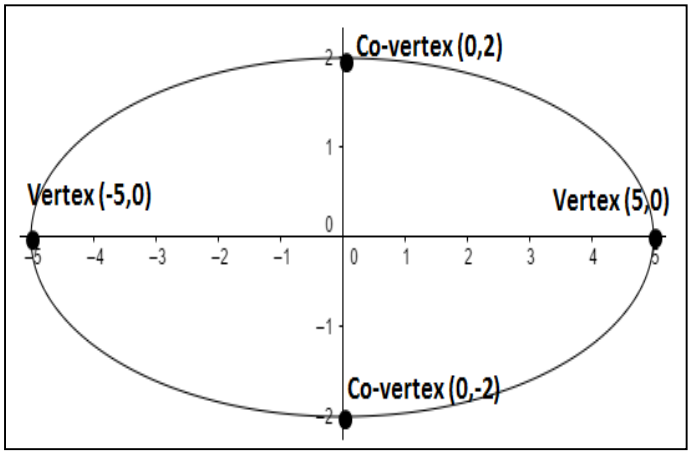
$$c = \sqrt{a^2 - b^2} = \sqrt{25 - 4} = \sqrt{21}$$

Center: $(h, k) = (0, 0)$

Vertices: $(h \pm a, k) = (\pm 5, 0)$

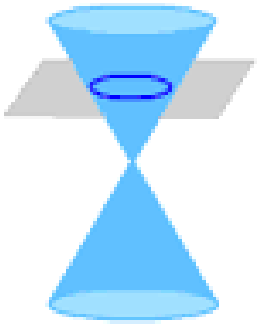
Co-vertices: $(h, k \pm b) = (0, \pm 2)$

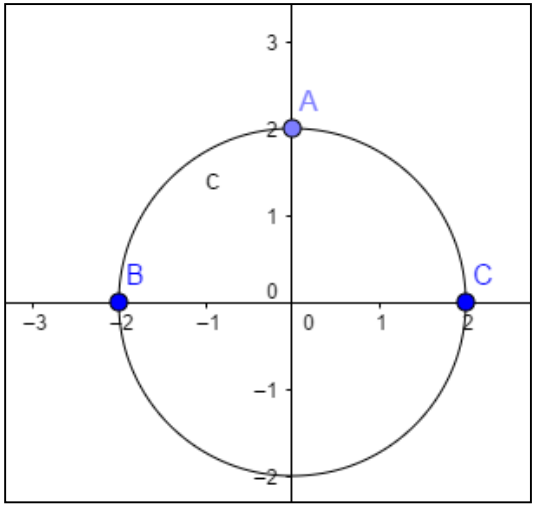
Foci: $(h \pm c, k) = \left(\pm\sqrt{21}, 0\right)$



$$4x^2 + 25y^2 = 100$$

A **Circle** is a curve formed by the intersection of a plane and a double cone such that the plane is perpendicular to the axis of cone.



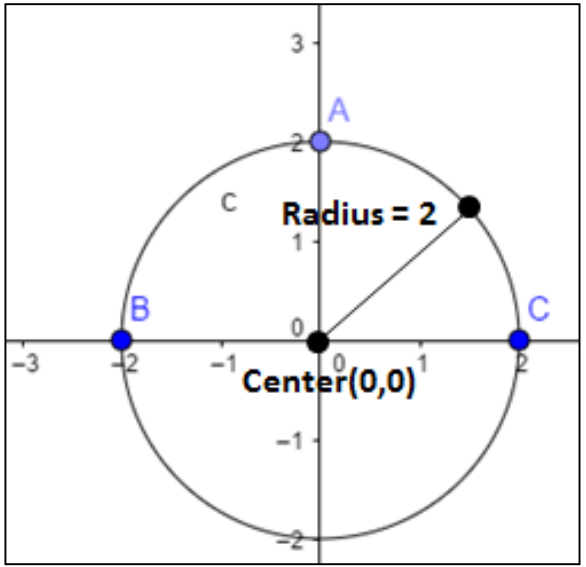


Circle

The equation of a circle is given as:

$$(x-h)^2+(y-k)^2=r^2$$

- (h,k) is the **center** of the circle and is the point that is equidistant from all the points on the circle.
- r is the radius of the circle and is the distance between the center and any point on the circle.



$$x^2+y^2=4$$

Problem 2: Graph $(x-2)^2+(y+1)^2=16$. Identify the center and radius of the circle.

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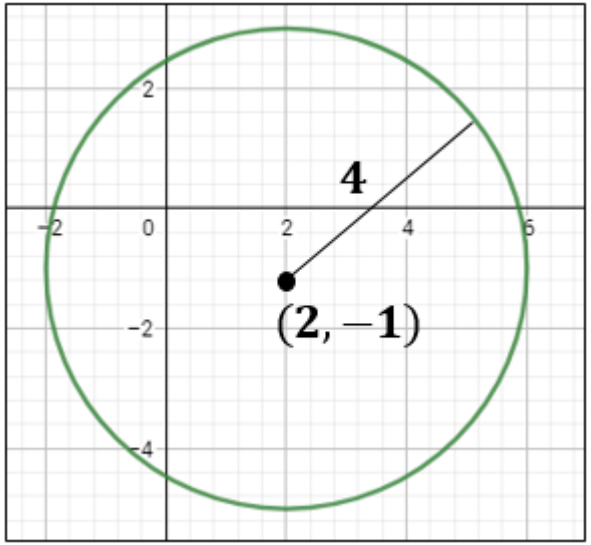
Comparing with the standard equation of circle:

$(x-h)^2+(y-k)^2=r^2 \leftrightarrow (x-2)^2+(y+1)^2=16$

$\rightarrow h=2\ ;\ k=-1\ ;\ r=4$

Center $(h,k)=(2,-1)$

Radius: $r=4$



$(x-2)^2+(y+1)^2=16$