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Matrix Multiplication, Inverses, and Determinants

Unit 6 Lesson 2

Students will be able to:

Understand matrix multiplication, determinants of matrices and their usage in finding inverse of a matrix.

Key Vocabulary:

- Matrix Multiplication
- Determinant of a Matrix
- Identity Matrix
- Inverse of a Matrix

Matrix Multiplication

If **A** and **B** are two matrices, then their multiplication is possible if the number of columns in matrix **A** is equal to the number of rows in the matrix **B**. If **A** has dimensions $m \times r$ and **B** has dimensions $r \times n$, then their product **AB** has dimensions $m \times n$.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then matrix multiplication AB is:

$$\boldsymbol{AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}}$$

Problem 1: Find AB if $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 4 \\ 2 & 1 & -1 \end{bmatrix}$.

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A has dimensions 2×2 and B has dimensions 2×3, so matrix multiplication is possible.

$$AB = \begin{bmatrix} 4(-1) + 2(2) & 4(2) + 2(1) & 4(4) + 2(-1) \\ 1(-1) + 3(2) & 1(2) + 3(1) & 1(4) + 3(-1) \end{bmatrix}$$

$$AB = \begin{bmatrix} -4 + 4 & 8 + 2 & 16 - 2 \\ -1 + 6 & 2 + 3 & 4 - 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 10 & 14 \\ 5 & 5 & 1 \end{bmatrix}$$

Determinant of a Matrix

The determinant of a matrix is the difference of the product of secondary diagonal entries from the main diagonal entries.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\mathbf{det}(\boldsymbol{A}) = \boldsymbol{ad} - \boldsymbol{cb}$

Identity Matrix

The identity matrix is a $n \times n$ matrix whose main diagonal has all entries equal to 1, and all other elements are 0s.

$$\boldsymbol{I}_{\mathbf{2\times2}} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Inverse of a Matrix

The inverse of a matrix $\boldsymbol{A}$ is a matrix $\boldsymbol{B}$, such that:

$$\boldsymbol{AB} = \boldsymbol{BA} = \boldsymbol{I}$$

Where I is the identity matrix.

Mathematically:

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\boldsymbol{A^{-1}} = \dfrac{\mathbf{1}}{\mathbf{det}(\boldsymbol{A})}\begin{bmatrix} \boldsymbol{d} & -\boldsymbol{b} \\ -\boldsymbol{c} & \boldsymbol{a} \end{bmatrix}$

where $\det(A) = ad - cb$

Problem 2: Find the inverse of matrix $A = \begin{bmatrix} 2 & -2 \\ 3 & -6 \end{bmatrix}$, if it exists.

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First find $\det(A)$ to see if inverse exists:

$$\mathbf{det}(A) = 2(-6) - 3(-2) = -12 + 6 = -6$$

Since $\det(A) \neq 0$, matrix A is invertible.

$$A^{-1} = \frac{1}{det(A)}\begin{bmatrix} -6 & 2 \\ -3 & 2 \end{bmatrix} \rightarrow A^{-1} = \frac{1}{-6}\begin{bmatrix} -6 & 2 \\ -3 & 2 \end{bmatrix}$$

$$\rightarrow A^{-1} = \begin{bmatrix} 1 & -\frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} \end{bmatrix}$$