

Matrix Multiplication, Inverses, and Determinants

 Guided Notes

Matrix Multiplication

If \mathbf{A} and \mathbf{B} are two matrices, then their multiplication is possible if the number of columns in matrix \mathbf{A} is equal to the number of rows in the matrix \mathbf{B} . If \mathbf{A} has dimensions $m \times r$ and \mathbf{B} has dimensions $r \times n$, then their product \mathbf{AB} has dimensions $m \times n$.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then matrix multiplication AB is:

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Problem 1: Find AB if $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 4 \\ 2 & 1 & -1 \end{bmatrix}$.

Determinant of a Matrix

The determinant of a matrix is the difference of the product of secondary diagonal entries from the main diagonal entries.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\det(A) = ad - cb$

Identity Matrix

The identity matrix is a $n \times n$ matrix whose main diagonal has all entries equal to 1, and all other elements are 0s.

$$I_{2 \times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Inverse of a Matrix

The inverse of a matrix A is a matrix B , such that:

$$AB = BA = I$$

Where I is the identity matrix.

Mathematically:

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

where $\det(A) = ad - cb$

Problem 2: Find the inverse of matrix $A = \begin{bmatrix} 2 & -2 \\ 3 & -6 \end{bmatrix}$, if it exists.