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Verifying Trigonometric Identities

Unit 5 Lesson 2

Students will be able to:

Understand how to verify trigonometric identities using reciprocal, quotient and Pythagorean identities

Key Vocabulary:

- Verifying Trigonometric Identities
- Reciprocal Identities
- Quotient Identities
- Pythagorean Identities

Verify Trigonometric Identity

To verify an identity means to prove both sides of the equation are equal for all values of the variable for which both sides are defined.

The process is to transform one side of the equation (usually the side with more complicated expressions) to the other side by simplifying using the algebra and trigonometric identities given below:

- **Reciprocal Identities**
- **Quotient Identities**
- **Pythagorean Identities**

Reciprocal Identities

Reciprocal identities relate these six identities such that one identity is the reciprocal of its co-identity.

Sine and Cosecant:

$$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{1}{\frac{hypotenuse}{opposite}} = \frac{1}{cosec(\theta)}$$

$$\sin(\theta) = \frac{1}{cosec(\theta)} \quad ; \quad cosec(\theta) = \frac{1}{\sin(\theta)}$$

Reciprocal Identities

Cosine and Secant:

$$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{1}{\frac{hypotenuse}{adjacent}} = \frac{1}{\sec(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)} \quad ; \quad \sec(\theta) = \frac{1}{\cos(\theta)}$$

Tangent and Cotangent:

$$\tan(\theta) = \frac{opposite}{adjacent} = \frac{1}{\frac{adjacent}{opposite}} = \frac{1}{\cot(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)} \quad ; \quad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Quotient Identities

Quotient identities relate the sine and cosine with tangent and cotangent of an angle in a right-angled triangle.

Tangent:

$$\tan(\theta) = \frac{opposite}{adjacent} = \frac{\frac{opposite}{hypotenuse}}{\frac{adjacent}{hypotenuse}} = \frac{\sin(\theta)}{\cos(\theta)}$$

Cotangent:

$$\cot(\theta) = \frac{adjacent}{opposite} = \frac{\frac{adjacent}{hypotenuse}}{\frac{opposite}{hypotenuse}} = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities

Pythagorean right-angled **identities** are written using the Pythagorean theorem for triangles.

$$1 = sin^2(\theta) + cos^2(\theta)$$

$$1 + tan^2(\theta) = sec^2(\theta)$$

$$1 + cot^2(\theta) = cosec^2(\theta)$$

Problem 1: Verify that $(sec^2\theta - 1)cos^2\theta = sin^2\theta$.

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Take the L.H.S:

$$(sec^2\theta - 1)cos^2\theta = tan^2\theta(cos^2\theta) \quad \text{(Pythagorean Identity)}$$

$$= \frac{sin^2\theta}{cos^2\theta} \times (cos^2\theta) \quad \text{(Quotient Identity)}$$

$$= sin^2\theta = \text{R.H.S}$$

$$\rightarrow (sec^2\theta - 1)cos^2\theta = sin^2\theta$$

Problem 2: Verify that $\frac{sec\theta}{sin\theta} - \frac{sin\theta}{cos\theta} = cot\theta$.

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Take the L.H.S:

$$\frac{sec\theta}{sin\theta} - \frac{sin\theta}{cos\theta} = \frac{\frac{1}{cos\theta}}{sin\theta} - \frac{sin^2\theta}{sin\theta cos\theta}$$ (Reciprocal Identity)

$$\frac{1}{sin\theta cos\theta} - \frac{sin^2\theta}{sin\theta cos\theta} = \frac{1 - sin^2\theta}{sin\theta cos\theta} = \frac{cos^2\theta}{sin\theta cos\theta}$$ (Pythagorean Identity)

$$\frac{cos^2\theta}{sin\theta cos\theta} = \frac{cos\theta}{sin\theta} = cot\theta$$ (Quotient Identity)

$$\rightarrow \frac{sec\theta}{sin\theta} - \frac{sin\theta}{cos\theta} = cot\theta$$