

Verify Trigonometric Identities Bell Work

Verify each identity.

$$1. \cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$$

$$2. \frac{1}{1-\tan^2\theta} + \frac{1}{1-\cot^2\theta} = 1$$

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Answers

Verify each identity.

1. $\cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$

Take L.H.S:

$$\cos^4 \theta - \sin^4 \theta = (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$$

$$= (\cos^2 \theta - \sin^2 \theta)(1) \quad (\text{Pythagorean Identity})$$

$$= \cos^2 \theta - \sin^2 \theta$$

= R.H.S

$$\rightarrow \cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$$

2. $\frac{1}{1-\tan^2 \theta} + \frac{1}{1-\cot^2 \theta} = 1$

Take L.H.S:

$$\frac{1}{1-\tan^2 \theta} + \frac{1}{1-\cot^2 \theta} = \frac{1-\cot^2 \theta+1-\tan^2 \theta}{(1-\tan^2 \theta)(1-\cot^2 \theta)}$$

$$= \frac{1 - \frac{\cos^2 \theta}{\sin^2 \theta} + 1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{\left(1 - \frac{\sin^2 \theta}{\cos^2 \theta}\right)\left(1 - \frac{\cos^2 \theta}{\sin^2 \theta}\right)} \quad (\text{Reciprocal Identity})$$

$$= \frac{\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}}{\left(\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}\right)\left(\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta}\right)}$$

$$= \frac{\frac{\cos^2 \theta (\sin^2 \theta - \cos^2 \theta) + \sin^2 \theta (\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta \sin^2 \theta}}{\left(\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}\right)\left(\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta}\right)}$$

$$= \frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{(\cos^2 \theta - \sin^2 \theta)} = (\cos^2 \theta + \sin^2 \theta) = 1 = \text{R.H.S} \quad (\text{Pythagorean Identity})$$

$$\rightarrow \frac{1}{1-\tan^2 \theta} + \frac{1}{1-\cot^2 \theta} = 1$$