

Name: _____ Period: _____ Date: _____

Verifying Trigonometric Identities Assignment

Verify each identity.

1. $\sec^2\theta(1 - \cos^2\theta) = \tan^2\theta$

2. $\tan\theta \operatorname{cosec}^2\theta - \tan\theta = \cot\theta$

3. $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

4. $\cos\theta - \cos\theta \cdot \sin^2\theta = \cos^3\theta$

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Verify each identity.

1. $\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

2. $\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$

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3. $\frac{1}{\operatorname{cosec}\theta+1} + \frac{1}{\operatorname{cosec}\theta-1} = 2\sec^2\theta\sin\theta$

4. $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

Verifying Trigonometric Identities Assignment

Answers

Verify each identity.

1. $\sec^2\theta(1 - \cos^2\theta) = \tan^2\theta$

Take L.H.S:

$$\sec^2\theta(1 - \cos^2\theta) = \sec^2\theta - \sec^2\theta\cos^2\theta$$

$$= \sec^2\theta - \frac{1}{\cos^2\theta}\cos^2\theta \quad (\text{Reciprocal Identity})$$

$$= \sec^2\theta - 1$$

$$= \tan^2\theta \quad (\text{Pythagorean Identity})$$

$$= \text{R.H.S}$$

$$\rightarrow \sec^2\theta(1 - \cos^2\theta) = \tan^2\theta$$

2. $\tan\theta\operatorname{cosec}^2\theta - \tan\theta = \cot\theta$

Take L.H.S:

$$\tan\theta\operatorname{cosec}^2\theta - \tan\theta = \tan\theta(\operatorname{cosec}^2\theta - 1)$$

$$= \tan\theta \cdot (\cot^2\theta) \quad (\text{Pythagorean Identity})$$

$$= \tan\theta \cdot \left(\frac{1}{\tan^2\theta}\right) \quad (\text{Reciprocal Identity})$$

$$= \frac{1}{\tan\theta} = \cot\theta \quad (\text{Reciprocal Identity})$$

$$= \text{R.H.S}$$

$$\rightarrow \tan\theta\operatorname{cosec}^2\theta - \tan\theta = \cot\theta$$

3. $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

Take L.H.S:

$$\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta} \quad (\text{Reciprocal Identity})$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1 - \sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta} \quad (\text{Pythagorean Identity})$$

$$\frac{\cos^2\theta}{\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta \quad (\text{Quotient Identity})$$

$$\rightarrow \frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$$

4. $\cos\theta - \cos\theta \cdot \sin^2\theta = \cos^3\theta$

Take L.H.S:

$$\cos\theta - \cos\theta \cdot \sin^2\theta = \cos\theta(1 - \sin^2\theta)$$

$$= \cos\theta(\cos^2\theta) \quad (\text{Pythagorean Identity})$$

$$= \cos^3\theta$$

$$= \text{R.H.S}$$

$$\rightarrow \cos\theta - \cos\theta \cdot \sin^2\theta = \cos^3\theta$$

Verifying Trigonometric Identities Assignment

Verify each identity.

$$1. \frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$$

Take L.H.S:

$$\frac{\sin\theta(\sin\theta)+(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)\sin\theta} = \frac{\sin^2\theta+(1-\cos\theta)^2}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin^2\theta+1+\cos^2\theta-2\cos\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{2-2\cos\theta}{\sin\theta(1-\cos\theta)} \quad (\text{Pythagorean Identity})$$

$$= \frac{2(1-\cos\theta)}{\sin\theta(1-\cos\theta)} = \frac{2}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta = \text{R.H.S} \quad (\text{Reciprocal Identity})$$

$$\rightarrow \frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$$

$$2. \frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$$

Take L.H.S:

$$\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \frac{\cos\theta+(1+\sin\theta)\tan\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta+(1+\sin\theta)\frac{\sin\theta}{\cos\theta}}{1+\sin\theta} \quad (\text{Reciprocal Identity})$$

$$= \frac{\cos\theta+\frac{\sin\theta}{\cos\theta}+\frac{\sin^2\theta}{\cos\theta}}{(1+\sin\theta)}$$

$$= \frac{\frac{\cos^2\theta+\sin\theta+\sin^2\theta}{\cos\theta}}{(1+\sin\theta)} = \frac{1+\sin\theta}{\cos\theta(1+\sin\theta)} \quad (\text{Pythagorean Identity})$$

$$= \frac{1}{\cos\theta} = \sec\theta = \text{R.H.S} \quad (\text{Reciprocal Identity})$$

$$\rightarrow \frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$$

Verifying Trigonometric Identities Assignment

$$3. \frac{1}{\operatorname{cosec}\theta+1} + \frac{1}{\operatorname{cosec}\theta-1} = 2\sec^2\theta\sin\theta$$

Take L.H.S:

$$\frac{1}{\operatorname{cosec}\theta+1} + \frac{1}{\operatorname{cosec}\theta-1} = \frac{\operatorname{cosec}\theta-1+\operatorname{cosec}\theta+1}{(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}$$

$$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}$$

$$= \frac{2\operatorname{cosec}\theta}{\cot^2\theta} \quad (\text{Pythagorean Identity})$$

$$= \frac{2}{\sin\theta} \times \frac{\sin^2\theta}{\cos^2\theta} \quad (\text{Pythagorean Identity})$$

$$= \frac{2\sin\theta}{\cos^2\theta} = 2\sec^2\theta\sin\theta = \text{R.H.S} \quad (\text{Reciprocal Identity})$$

$$\rightarrow \frac{1}{\operatorname{cosec}\theta+1} + \frac{1}{\operatorname{cosec}\theta-1} = 2\sec^2\theta\sin\theta$$

$$4. \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$$

Take L.H.S:

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = \frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta} \quad (\text{Pythagorean Identity})$$

$$= 2\sec^2\theta = \text{R.H.S} \quad (\text{Reciprocal Identity})$$

$$\rightarrow \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$$