**Rational Functions** Guided Notes

**Rational Function**

A rational function is a fraction of polynomials \( p(x) \) and \( q(x) \).

**Mathematically:**

\[
 f = \frac{p(x)}{q(x)}
\]

Where \( p(x) \) is the numerator and \( q(x) \) is the denominator.

**Example:**

- \( f = \frac{1}{x-1} \)
- \( f = \frac{y(y-2)}{(y-3)} \)

**Domain of Rational Function**

Domain of a rational function is the set of all real numbers except the roots of the denominator polynomial \( q(x) \).

**Range of Rational Function**

Range of a rational function is the set of all real numbers except those values of input (domain) that give the output as \( \infty \) i.e. the output of the numbers excluded from domain.

**Vertical Asymptotes**

Vertical asymptotes are the vertical lines passing through the roots of denominator polynomial \( q(x) \) and touching the graph of the rational function. The graph of rational function rises up or slides down the sides of the vertical asymptotes.

**x-intercepts**

These are the points where the graph of a rational function meets the \( x \)-axis and are the roots of the numerator polynomial \( f(x) \) in the rational function.
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Problem 1: Find the domain, vertical asymptotes and x-intercepts of the rational function \( \frac{4(x-2)(x^2-1)}{3(x-3)(x+4)} \).

Interpreting Graph of Rational Function

Given the graph of a rational function, we can identify its vertical asymptotes and x-intercepts.

From the graph we can see that the function has two vertical asymptotes at \( x = -3 \) and \( x = -1 \) (since \( f \) is \( \infty \)).

Also, the graph touches x-axis at \( x = -4 \) and \( x = -2 \), which are the x-intercepts.

We can also say that numerator \( p(x) \) has factors \((x+4)(x+2)\) and denominator \( q(x) \) has factors \((x+3)(x+2)\).

Problem 2: Solve the equation \( y + \frac{6}{y} = 5 \).