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Inverse Relations and Functions Guided Notes

The inverse of a relation is a relation obtained by reversing or swapping the coordinates of each ordered pair in the relation.

If the relation is described by an equation in the variables x and y, the equation of the inverse relation is obtained by replacing every x in the equation with y and every y in the equation with x.

If $f(x)$ represents a function of x, the inverse of the function is represented by the symbol $f^{-1}(x)$. $f^{-1}(x) \neq \frac{1}{f(x)}$.

Sample Problem 1: Find the inverse of each relation given as a set of ordered pairs.

a.

x	0	1	2	4
y	-2	-3	-5	-1

x	-2	-3	-5	-1
y	0	1	2	4

b.

x	-2	-5	-6	1
y	-6	-10	-3	-9

x	-6	-10	-3	-9
y	-2	-5	-6	1

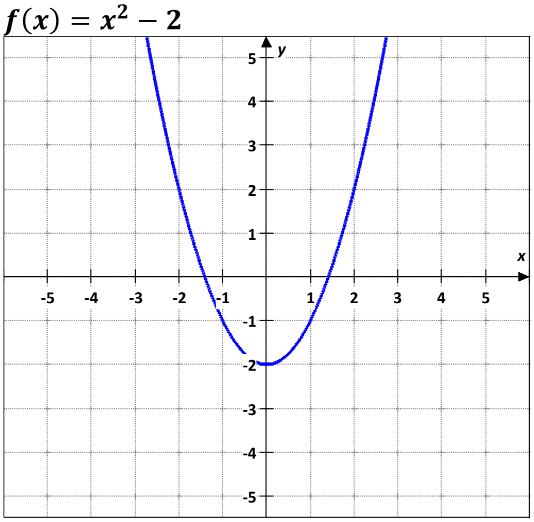
Horizontal Line Test

A function f has an inverse function f^{-1} if and only if each horizontal line intersects the graph of the function in at most one point.
If a function passes the horizontal line test, then it is said to be one-to-one, because no x-value is matched with more than one y-value and no y-value is matched with more than one x -value.

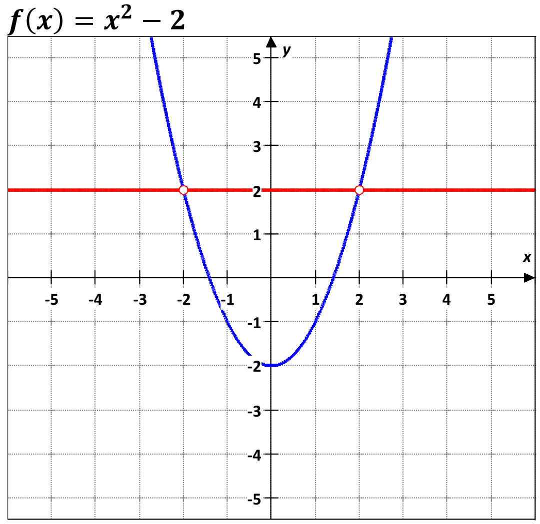
If a function f is one-to-one, it has an inverse function f^{-1} such that the domain of f is equal to the range of f^{-1}, and the range of f is equal to the domain of f^{-1}.

Sample Problem 2: Use a horizontal line test to determine whether of the graph of each function is a one-to-one function.

a. $f(x) = x^2 - 2$



$f(x) = x^2 - 2$



Because we can draw at least one horizontal line that intersects the graph more than once, $f(x) = x^2 - 2$ is not a one-to-one function. Therefore the inverse of $f(x) = x^2 - 2$ is a relation, but not a function.

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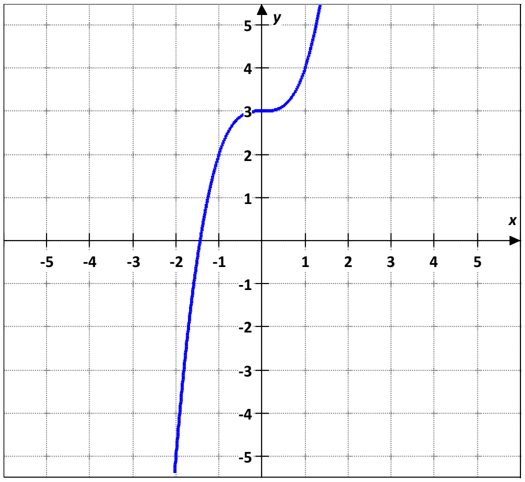
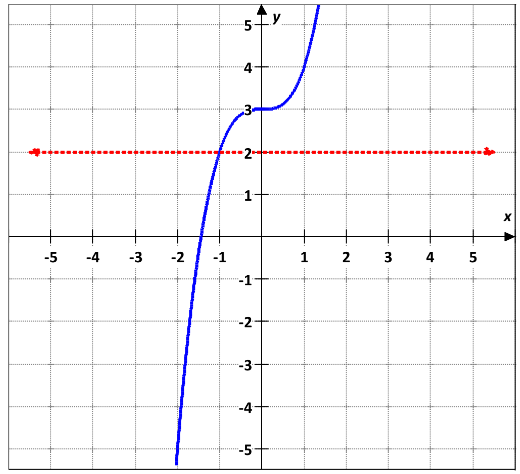
b. $f(x) = x^3 + 3$ $f(x) = x^3 + 3$





Because every horizontal line we draw intersects the graph only once, $f(x) = x^3 + 3$ is a one-to-one function. Therefore the inverse of $f(x) = x^3 + 3$ is a function.

Step by Step Procedure to Find the Inverse of $f(x)$

1. Determine whether the function has an inverse by checking to see if it is one-to-one.
2. Replace $f(x)$ with y.
3. Interchange x and y.
4. Solve for y.
5. Replace y with $f^{-1}(x)$.
6. State any restrictions on the domain of $f^{-1}(x)$. Sometimes only part of the function you find algebraically may be the inverse function of f. Therefore, be sure to analyze the domain of f when finding f^{-1}.

The graphs of the function and the inverse function are reflections across the line $y = x$.

Sample Problem 3: Find the inverse function, state any restrictions on its domain and then graph the function and its inverse.

a. $f(x) = 4x - 3$

$f(x) = 4x - 3 \quad D = (-\infty, \infty) \quad R = (-\infty, \infty)$

$y = 4x - 3$

$x = 4y - 3$

$x + 3 = 4y - 3 + 3$

$x + 3 = 4y$

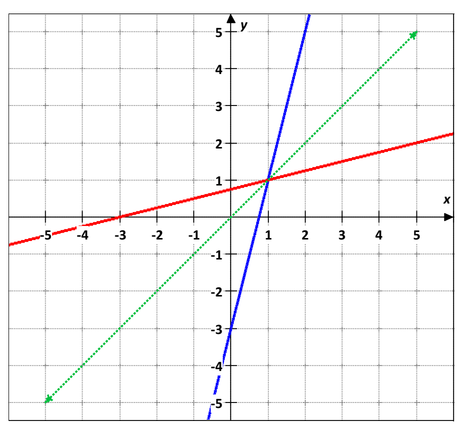
$\dfrac{x+3}{4} = y$

$f^{-1}(x) = \dfrac{x+3}{4}$

$D = (-\infty, \infty) \quad R = (-\infty, \infty)$

$f(x) = 4x - 3$

$f^{-1}(x) = \dfrac{x+3}{4}$



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b. $f(x) = (x-2)^3$

$f(x) = (x-2)^3 \quad D = (-\infty, \infty) \quad R = (-\infty, \infty)$

$y = (x-2)^3$

$x = (y-2)^3$

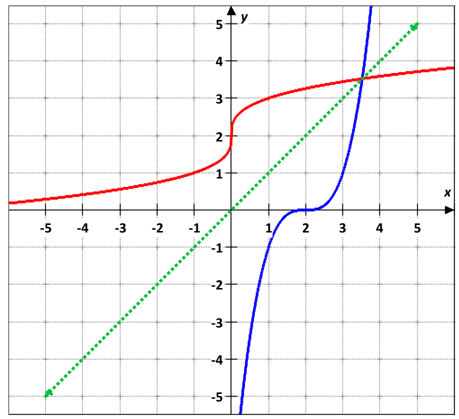
$\sqrt[3]{x} = \sqrt[3]{(y-2)^3}$

$\sqrt[3]{x} = y - 2$

$\sqrt[3]{x} + 2 = y - 2 + 2$

$\sqrt[3]{x} + 2 = y$

$f^{-1}(x) = \sqrt[3]{x} + 2 \quad D = (-\infty, \infty) \quad R = (-\infty, \infty)$



$f(x) = (x-2)^3$ →

$f^{-1}(x) = \sqrt[3]{x} + 2$ →

c. $f(x) = 2\sqrt{x+1}$

$f(x) = 2\sqrt{x+1} \quad D = [-1, \infty) \quad R = [0, \infty)$

$y = 2\sqrt{x+1}$

$x = 2\sqrt{y+1}$

$x^2 = 2^2\left(\sqrt{y+1}\right)^2$

$x^2 = 4(y+1)$

$x^2 = 4y + 4$

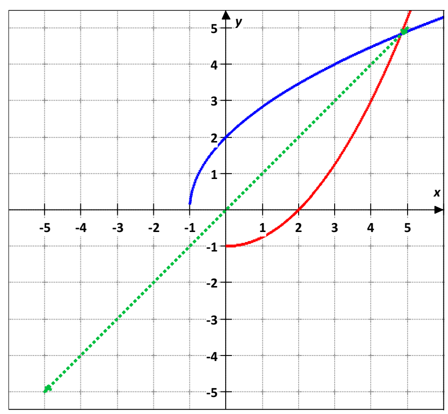
$x^2 - 4 = 4y + 4 - 4$

$x^2 - 4 = 4y$

$\dfrac{x^2 - 4}{4} = y$

$f^{-1}(x) = \dfrac{x^2 - 4}{4} \quad x \geq 0$

$D = [0, \infty) \quad R = [-1, \infty)$



$f(x) = 2\sqrt{x+1}$ →

$f^{-1}(x) = \dfrac{x^2 - 4}{4} \quad x \geq 0$ →

Compositions of Inverse Functions

Two functions, f and g, are inverse functions if and only if:

- $f(g(x)) = x$, for every x in the domain of $g(x)$ and
- $g(f(x)) = x$, for every x in the domain of $f(x)$.

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Sample Problem 4: Show algebraically that f and g are inverse functions.

a.

$$f(x) = 7x - 4$$
$$g(x) = \frac{x+4}{7}$$
$$f(g(x)) = 7(g(x)) - 4$$
$$f(g(x)) = 7\left(\frac{x+4}{7}\right) - 4$$
$$f(g(x)) = x + 4 - 4$$
$$f(g(x)) = x$$

$$g(f(x)) = \frac{f(x)+4}{7}$$
$$g(f(x)) = \frac{7x-4+4}{7}$$
$$g(f(x)) = \frac{7x}{7}$$
$$g(f(x)) = x$$

b.

$$f(x) = \sqrt{x+5} - 3$$
$$g(x) = x^2 + 6x + 4 \qquad x \geq 3$$

$$f(g(x)) = \sqrt{g(x)+5} - 3$$
$$f(g(x)) = \sqrt{x^2+6x+4+5} - 3$$
$$f(g(x)) = \sqrt{x^2+6x+9} - 3$$
$$f(g(x)) = \sqrt{(x+3)^2} - 3$$
$$f(g(x)) = (x+3) - 3$$
$$f(g(x)) = x$$

$$g(f(x)) = (f(x))^2 + 6 * f(x) + 4$$
$$g(f(x)) = (\sqrt{x+5} - 3)^2 + 6 * (\sqrt{x+5} - 3) + 4$$
$$g(f(x)) = x + 5 - 6\sqrt{x+5} + 9 + 6\sqrt{x+5} - 18 + 4$$
$$g(f(x)) = x$$