The inverse of a relation is a relation obtained by reversing or swapping the coordinates of each ordered pair in the relation.

If the relation is described by an equation in the variables and the equation of the inverse relation is obtained by replacing every in the equation with and every in the equation with .

If represents a function of, the inverse of the function is represented by the symbol . **.**

**Sample Problem 1: Find the inverse of each relation given as a set of ordered pairs.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a.** | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |
| **b.** | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |

**Horizontal Line Test**

A function has an inverse function if and only if each horizontal line intersects the graph of the function in at most one point.If a function passes the horizontal line test, then it is said to be one-to-one, because no -value is matched with more than one -value and no -value is matched with more than one -value.

If a function is one-to-one, it has an inverse function such that the domain of is equal to the range of, and the range of is equal to the domain of.

**Sample Problem 2: Use a horizontal line test to determine whether of the graph of each function is a one-to-one function.**

|  |  |  |
| --- | --- | --- |
| **a.** |  |  |
|  | Because we can draw at least one horizontal line that intersects the graph more than once, is not a one-to-one function. Therefore the inverse of is a relation, but not a function. | |
| **b.** |  |  |
|  | Because every horizontal line we draw intersects the graph only once, is a one-to-one function.  Therefore the inverse of is a function. | |

**Step by Step Procedure to Find the Inverse of**

1. Determine whether the function has an inverse by checking to see if it is one-to-one.
2. Replace with
3. Interchange and
4. Solve for
5. Replace with
6. State any restrictions on the domain of . Sometimes only part of the function you find algebraically may be the inverse function of. Therefore, be sure to analyze the domain of when finding

The graphs of the function and the inverse function are reflections across the line

**Sample Problem 3: Find the inverse function, state any restrictions on its domain and then graph the function and its inverse.**

|  |  |  |
| --- | --- | --- |
| **a.** |  |  |
|  |  |  |
| **b.** |  |  |
|  |  |  |
| **c.** |  |  |
|  |  |  |

**Compositions of Inverse Functions**

Two functions, and, are inverse functions if and only if:

* for every in the domain of and
* for every in the domain of .

**Sample Problem 4:**  **Show algebraically that**  **and are inverse functions.**

|  |  |  |  |
| --- | --- | --- | --- |
| **a.** |  | **b.** |  |
|  |  |  |  |