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Inverse Relations and Functions Guided Notes

The inverse of a relation is a relation obtained by reversing or swapping the coordinates of each ordered pair in the relation.

If the relation is described by an equation in the variables x and y, the equation of the inverse relation is obtained by replacing every x in the equation with y and every y in the equation with x.

If $f(x)$ represents a function of x, the inverse of the function is represented by the symbol $f^{-1}(x)$. $f^{-1}(x) \neq \frac{1}{f(x)}$.

Sample Problem 1: Find the inverse of each relation given as a set of ordered pairs.

a.

x	0	1	2	4
y	-2	-3	-5	-1

x				
y				

b.

x	-2	-5	-6	1
y	-6	-10	-3	-9

x				
y				

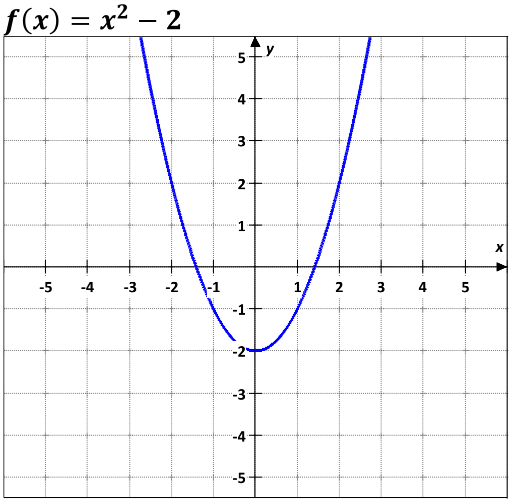
Horizontal Line Test

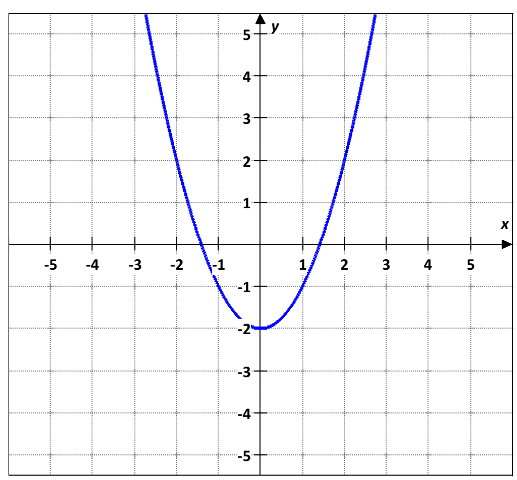
A function f has an inverse function f^{-1} if and only if each horizontal line intersects the graph of the function in at most one point.
If a function passes the horizontal line test, then it is said to be one-to-one, because no x-value is matched with more than one y-value and no y-value is matched with more than one x -value.

If a function f is one-to-one, it has an inverse function f^{-1} such that the domain of f is equal to the range of f^{-1}, and the range of f is equal to the domain of f^{-1}.

Sample Problem 2: Use a horizontal line test to determine whether of the graph of each function is a one-to-one function.

a. $f(x) = x^2 - 2$





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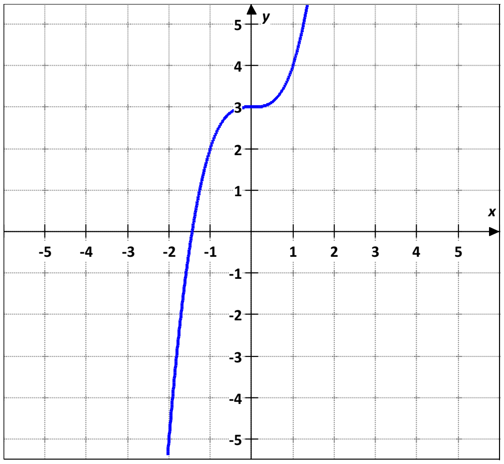
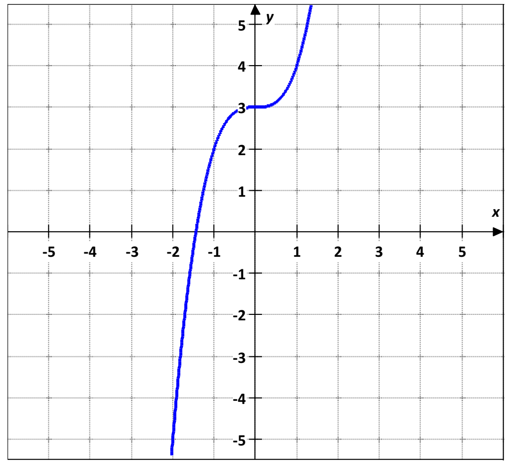
Inverse Relations and Functions Guided Notes

b. $f(x) = x^3 + 3$





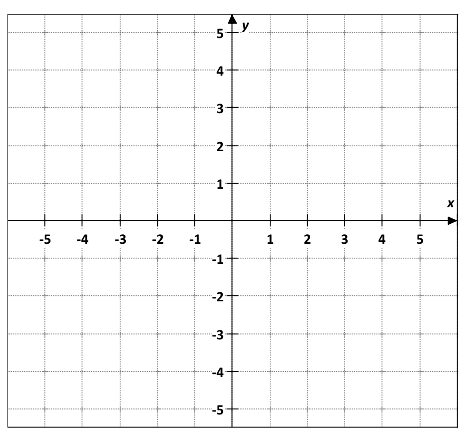
Step by Step Procedure to Find the Inverse of $f(x)$

1. Determine whether the function has an inverse by checking to see if it is one-to-one.
2. Replace $f(x)$ with y.
3. Interchange x and y.
4. Solve for y.
5. Replace y with $f^{-1}(x)$.
6. State any restrictions on the domain of $f^{-1}(x)$. Sometimes only part of the function you find algebraically may be the inverse function of f. Therefore, be sure to analyze the domain of f when finding f^{-1}.

The graphs of the function and the inverse function are reflections across the line $y = x$.

Sample Problem 3: **Find the inverse function, state any restrictions on its domain and then graph the function and its inverse.**

a. $f(x) = 4x - 3$



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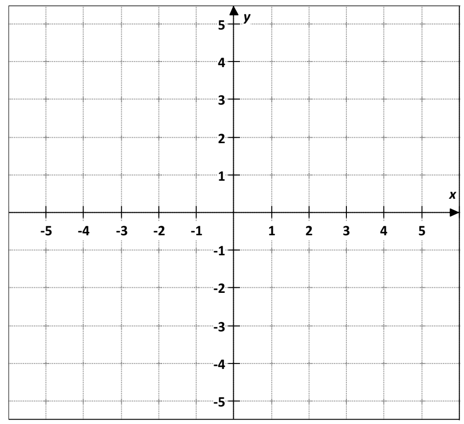
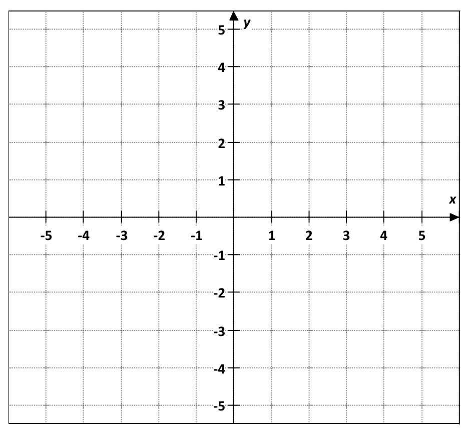
Inverse Relations and Functions Guided Notes

b. $f(x) = (x-2)^3$



c. $f(x) = 2\sqrt{x+1}$



Compositions of Inverse Functions

Two functions, f and g, are inverse functions if and only if:

- $f(g(x)) = x$, for every x in the domain of $g(x)$ and
- $g(f(x)) = x$, for every x in the domain of $f(x)$.

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Inverse Relations and Functions Guided Notes

Sample Problem 4: Show algebraically that f and g are inverse functions.

a. $f(x) = 7x - 4$

$g(x) = \frac{x+4}{7}$

b. $f(x) = \sqrt{x+5} - 3$

$g(x) = x^2 + 6x + 4 \qquad x \geq 3$