**Find the inverse of each relation.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1.** | $$y=4x$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$1$$ | $$-1$$ | $$2$$ | $$-2$$ |
| $$y$$ | $$4$$ | $$-4$$ | $$8$$ | $$-8$$ |

 | **2.** | $$y=2x^{2}-3$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0$$ | $$1$$ | $$2$$ | $$-1$$ |
| $$y$$ | $$-3$$ | $$-1$$ | $$5$$ | $$-1$$ |

 |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$y$$ |  |  |  |  |

 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$y$$ |  |  |  |  |

 |
| **3.** | $$y=\frac{x}{2}$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$2$$ | $$4$$ | $$6$$ | $$8$$ |
| $$y$$ | $$1$$ | $$2$$ | $$3$$ | $$4$$ |

 | **4.** | $$y=\left|2x\right|$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0$$ | $$1$$ | $$2$$ | $$-1$$ |
| $$y$$ | $$0$$ | $$2$$ | $$4$$ | $$2$$ |

 |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$y$$ |  |  |  |  |

 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$y$$ |  |  |  |  |

 |

**Use a horizontal line test to determine whether of the graph of each function is a one-to-one function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **5.** | $$f\left(x\right)=\frac{5x}{7}$$ | **6.** | $$f\left(x\right)=x^{2}+2x-2 $$ |
|  |  |  |  |
| **7.** | $$h\left(x\right)=2-\frac{1}{x}$$ | **8.** | $$g\left(x\right)=\sqrt{2x+3}$$ |
|  |  |  |  |

**Find an equation for the inverse of each of the one to one function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **9.** | $$f\left(x\right)=6x-4 $$ | **10.** | $$f\left(x\right)=5x^{3}+2 $$ |
|  | $$ $$ |  |  |

**Use the graph of each function to graph its inverse function.**

|  |  |  |
| --- | --- | --- |
| **13.** | $$f\left(x\right)=3x+1$$ |  |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ |
| $$y$$ | $$-2$$ | $$1$$ | $$4$$ | $$7$$ |

 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$y$$ |  |  |  |  |

 |
| **14.** | $$f\left(x\right)=2x^{3}-4$$ |  |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ |
| $$y$$ | $$-6$$ | $$-4$$ | $$-2$$ | $$12$$ |

 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$y$$ |  |  |  |  |

 |

**Determine whether each function has an inverse function. If it does, find the inverse function and state any restrictions on its domain.**

|  |  |  |
| --- | --- | --- |
| **15.** | $$f\left(x\right)=\frac{4x+1}{x-1} $$ |  |
|  |  |  |
| **16.** | $$f\left(x\right)=-x^{3}+2$$ |  |
|  |  |  |

**Show algebraically that** $f$ **and**$ g$ **are inverse functions.**

|  |  |  |  |
| --- | --- | --- | --- |
| **17.**  | $$f\left(x\right)=\frac{x+2}{3} g\left(x\right)=3x-2$$ | **18.** | $f\left(x\right)=\sqrt{x-1}+3$$$g\left(x\right)=\left(x-3\right)^{2}+1 x\geq 3$$ |
|  |  |  |  |
| **19.**  | $$f\left(x\right)=\frac{2}{x-3} g\left(x\right)=\frac{2+3x}{x}$$ | **20.** | $$f\left(x\right)=\frac{2x-1}{x-1} g\left(x\right)=\frac{x-1}{x-2}$$ |
|  |   |  |  |

**ANSWERS**

**Find the inverse of each relation.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1.** | $$y=4x$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$1$$ | $$-1$$ | $$2$$ | $$-2$$ |
| $$y$$ | $$4$$ | $$-4$$ | $$8$$ | $$-8$$ |

 | **2.** | $$y=2x^{2}-3$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0$$ | $$1$$ | $$2$$ | $$-1$$ |
| $$y$$ | $$-3$$ | $$-1$$ | $$5$$ | $$-1$$ |

 |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$4$$ | $$-4$$ | $$8$$ | $$-8$$ |
| $$y$$ | $$1$$ | $$-1$$ | $$2$$ | $$-2$$ |

 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-3$$ | $$-1$$ | $$5$$ | $$-1$$ |
| $$y$$ | $$0$$ | $$1$$ | $$2$$ | $$-1$$ |

 |
| **3.** | $$y=\frac{x}{2}$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$2$$ | $$4$$ | $$6$$ | $$8$$ |
| $$y$$ | $$1$$ | $$2$$ | $$3$$ | $$4$$ |

 | **4.** | $$y=\left|2x\right|$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0$$ | $$1$$ | $$2$$ | $$-1$$ |
| $$y$$ | $$0$$ | $$2$$ | $$4$$ | $$2$$ |

 |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$1$$ | $$2$$ | $$3$$ | $$4$$ |
| $$y$$ | $$2$$ | $$4$$ | $$6$$ | $$8$$ |

 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0$$ | $$2$$ | $$4$$ | $$2$$ |
| $$y$$ | $$0$$ | $$1$$ | $$2$$ | $$-1$$ |

 |

**Use a horizontal line test to determine whether of the graph of each function is a one-to-one function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **5.** | $$f\left(x\right)=\frac{5x}{7}$$ | **6.** | $$f\left(x\right)=x^{2}+2x-2 $$ |
|  | $$f\left(x\right)=\frac{5x}{7} is a one-to-one function. $$ |  | $f\left(x\right)=x^{2}+2x-2 is not a one-to-one function. $  |
| **7.** | $$h\left(x\right)=2-\frac{1}{x}$$ | **8.** | $$g\left(x\right)=\sqrt{2x+3}$$ |
|  | $$h\left(x\right)=2-\frac{1}{x} is a one-to-one function. $$ |  | $g\left(x\right)=\sqrt{2x+3} is a one-to-one function. $  |

**Find an equation for the inverse of each of the one to one function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **9.** | $$f\left(x\right)=6x-4 $$ | **10.** | $$f\left(x\right)=5x^{3}+2 $$ |
|  | $$f\left(x\right)=6x-4 $$$$y=6x-4$$$$x=6y-4$$$$x+4=6y-4+4$$$$x+4=6y$$$$\frac{x+4}{6}=y$$$$f^{-1}\left(x\right)=\frac{x+4}{6} $$ |  | $$f\left(x\right)=5x^{3}+2$$$$y=5x^{3}+2$$$$x=5y^{3}+2$$$$x-2=5y^{3}+2-2$$$$x-2=5y^{3}$$$$\frac{x-2}{5}=y^{3}$$$$\sqrt[3]{\frac{x-2}{5}}=y$$$$f^{-1}\left(x\right)=\sqrt[3]{\frac{x-2}{5}} $$ |

**Use the graph of each function to graph its inverse function.**

|  |  |  |
| --- | --- | --- |
| **13.** | $$f\left(x\right)=3x+1$$ |  |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ |
| $$y$$ | $$-2$$ | $$1$$ | $$4$$ | $$7$$ |

 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-2$$ | $$1$$ | $$4$$ | $$7$$ |
| $$y$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ |

 |
| **14.** | $$f\left(x\right)=2x^{3}-4$$ |  |
|  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ |
| $$y$$ | $$-6$$ | $$-4$$ | $$-2$$ | $$12$$ |

 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-6$$ | $$-4$$ | $$-2$$ | $$12$$ |
| $$y$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ |

 |

**Determine whether each function has an inverse function. If it does, find the inverse function and state any restrictions on its domain.**

|  |  |  |
| --- | --- | --- |
| **15.** | $$f\left(x\right)=\frac{4x+1}{x-1} $$ |  |
|  | $$f\left(x\right)=\frac{4x+1}{x-1} is a one-to-one function. $$Therefore the inverse of $f\left(x\right)=\frac{4x+1}{x-1}$ is a function. | $$f\left(x\right)=\frac{4x+1}{x-1} x\ne 1 $$$$D=\left(-\infty ,1\right)∪\left(1,\infty \right) R=\left(-\infty ,4\right)∪\left(4,\infty \right)$$$$ $$$$y=\frac{4x+1}{x-1}$$$$x=\frac{4y+1}{y-1}$$$$x\left(y-1\right)=4y+1$$$$xy-x=4y+1$$$$xy-4y=x+1$$$$y(x-4)=x+1$$$$y=\frac{x+1}{x-4}$$$$f^{-1}\left(x\right)=\frac{x+1}{x-4} x\ne 4$$$D= (-\infty ,4)∪(4,\infty )$$ R=(-\infty ,1)∪(1,\infty )$ |
| **16.** | $$f\left(x\right)=-x^{3}+2$$ |  |
|  | $f\left(x\right)=-x^{3}+2$$ is a one-to-one function.$Therefore the inverse of$ f\left(x\right)=-x^{3}+2$ is a function. | $f\left(x\right)=-x^{3}+2 $$ D=(-\infty ,\infty ) R= (-\infty ,\infty )$$$y=-x^{3}+2 $$$$x=-y^{3}+2 $$$$x-2=-y^{3}+2-2$$$$x-2=-y^{3}$$$$y^{3}=2-x$$$$y=\sqrt[3]{2-x}$$$f^{-1}\left(x\right)=\sqrt[3]{2-x}$$ D=(-\infty ,\infty ) R= (-\infty ,\infty )$ |

**Show algebraically that** $f$ **and**$ g$ **are inverse functions.**

|  |  |  |  |
| --- | --- | --- | --- |
| **17.**  | $$f\left(x\right)=\frac{x+2}{3} g\left(x\right)=3x-2$$ | **18.** | $f\left(x\right)=\sqrt{x-1}+3$$$g\left(x\right)=\left(x-3\right)^{2}+1 x\geq 3$$ |
|  | $$f\left(g\left(x\right)\right)=\frac{g\left(x\right)+2}{3}$$$$f\left(g\left(x\right)\right)=\frac{3x-2+2}{3}$$$$f\left(g\left(x\right)\right)=\frac{3x}{3}$$$$f\left(g\left(x\right)\right)=x$$$$g\left(f\left(x\right)\right)=3\*f\left(x\right)-2$$$$g\left(f\left(x\right)\right)=3\*\frac{x+2}{3}-2$$$$g\left(f\left(x\right)\right)=x+2-2$$$$g\left(f\left(x\right)\right)=x$$ |  | $$f\left(g\left(x\right)\right)=\sqrt{g\left(x\right)-1}+3$$$$f\left(g\left(x\right)\right)=\sqrt{\left(x-3\right)^{2}+1-1}+3$$$$f\left(g\left(x\right)\right)=\sqrt{\left(x-3\right)^{2}}+3$$$$f\left(g\left(x\right)\right)=\left(x-3\right)+3$$$$f\left(g\left(x\right)\right)=x$$$$g\left(f\left(x\right)\right)=\left(f\left(x\right)-3\right)^{2}+1$$$$g\left(f\left(x\right)\right)=\left(\sqrt{x-1}+3-3\right)^{2}+1$$$$g\left(f\left(x\right)\right)=\left(\sqrt{x-1}\right)^{2}+1$$$$g\left(f\left(x\right)\right)=x-1+1$$$$\left(f\left(x\right)\right)=x$$ |
| **19.**  | $$f\left(x\right)=\frac{2}{x-3} g\left(x\right)=\frac{2+3x}{x}$$ | **20.** | $$f\left(x\right)=\frac{2x-1}{x-1} g\left(x\right)=\frac{x-1}{x-2}$$ |
|  |  $$f\left(g\left(x\right)\right)=\frac{2}{g\left(x\right)-3}$$$$f\left(g\left(x\right)\right)=\frac{2}{\frac{2+3x}{x}-3}$$$$f\left(g\left(x\right)\right)=\frac{2}{\frac{2+3x-3x}{x}}$$$$f\left(g\left(x\right)\right)=\frac{2}{\frac{2}{x}}$$$$f\left(g\left(x\right)\right)=x$$$$g\left(f\left(x\right)\right)=\frac{2+3\*f\left(x\right)}{f\left(x\right)}$$$$g\left(f\left(x\right)\right)=\frac{2+3\*\frac{2}{x-3}}{\frac{2}{x-3}}$$$$g\left(f\left(x\right)\right)=\frac{\frac{2x-6+6}{x-3}}{\frac{2}{x-3}}=\frac{2x}{2}$$$$g\left(f\left(x\right)\right)=\frac{\frac{2x}{x-3}}{\frac{2}{x-3}}=\frac{2x}{2}$$$$g\left(f\left(x\right)\right)=x$$ |  | $$f\left(g\left(x\right)\right)=\frac{2\*g\left(x\right)-1}{g\left(x\right)-1}$$$$f\left(g\left(x\right)\right)=\frac{2\*\frac{x-1}{x-2}-1}{\frac{x-1}{x-2}-1}$$$$f\left(g\left(x\right)\right)=\frac{\frac{2x-2-x+2}{x-2}}{\frac{x-1-x+2}{x-2}}$$$$f\left(g\left(x\right)\right)=\frac{\frac{x}{x-2}}{\frac{1}{x-2}}$$$$f\left(g\left(x\right)\right)=x$$$$g\left(f\left(x\right)\right)=\frac{f\left(x\right)-1}{f\left(x\right)-2}$$$$g\left(f\left(x\right)\right)=\frac{\frac{2x-1}{x-1}-1}{\frac{2x-1}{x-1}-2}$$$$g\left(f\left(x\right)\right)=\frac{\frac{2x-1-x+1}{x-1}}{\frac{2x-1-2x+2}{x-1}}$$$$g\left(f\left(x\right)\right)=\frac{\frac{x}{x-1}}{\frac{1}{x-1}}$$$$g\left(f\left(x\right)\right)=x$$ |