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Continuity, End Behavior, and Limits Guided Notes

The graph of **a continuous function** has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.

One condition for a function $\boldsymbol{f(x)}$ to be continuous at $\boldsymbol{x = c}$ is that the function must approach a unique function value as $\boldsymbol{x}$ -values approach $\boldsymbol{c}$ from the left and right sides. The concept of approaching a value without necessarily ever reaching it is called **a limit.**

If the value of $\boldsymbol{f(x)}$approaches a unique value $\boldsymbol{L}$ as $\boldsymbol{x}$ approaches $\boldsymbol{c}$ from each side, then the limit of $\boldsymbol{f(x)}$ as $\boldsymbol{x}$ approaches $\boldsymbol{c}$ is $\boldsymbol{L}$. $\boldsymbol{\lim_{x \to c} f(x) = L}$

Functions that are not continuous are discontinuous. Graphs that are discontinuous can exhibit:

- Infinite discontinuity (A function has an infinite discontinuity at $\boldsymbol{x = c}$ if the function value increases or decreases indefinitely as $\boldsymbol{x}$ approaches $\boldsymbol{c}$ from the left and right)
- Jump discontinuity,(A function has a jump discontinuity at $\boldsymbol{x = c}$ if the limits of the function as $\boldsymbol{x}$ approaches $\boldsymbol{c}$ from the left and right exist but have two distinct values.
- Removable discontinuity, also called point discontinuity (function has a removable discontinuity if the function is continuous everywhere except for a hole at $\boldsymbol{x = c}$.

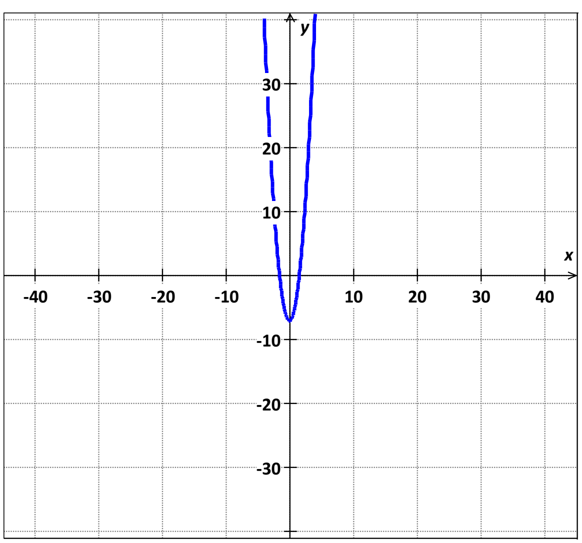
Continuity Test

A function $\boldsymbol{f(x)}$ is continuous at $\boldsymbol{x = c}$ if it satisfies the following conditions**.**

1. $\boldsymbol{f(x)}$ is defined at **c.** $\boldsymbol{f(c)}$exists.
2. $\boldsymbol{f(x)}$ approaches the same function value to the left and right of $\boldsymbol{c}$. $\boldsymbol{\lim_{x \to c} f(x)\ exists}$
3. The function value that $\boldsymbol{f(x)}$approaches from each side of $\boldsymbol{c}$ is $\boldsymbol{f(c)}$. $\boldsymbol{\lim_{x \to c} f(x) = f\ (c)}$

Sample Problem 1: Determine whether each function is continuous at the given x -values. Justify using the continuity test. If discontinuous, identify the type of discontinuity.

a. $\boldsymbol{f(x) = 3x^2 + x - 7 \quad at\ \ x = 1}$



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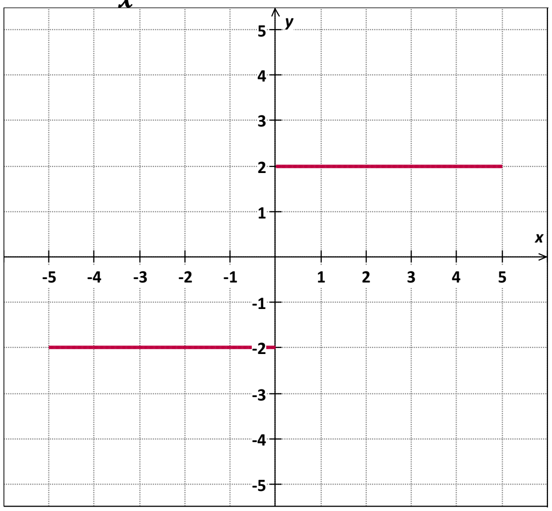
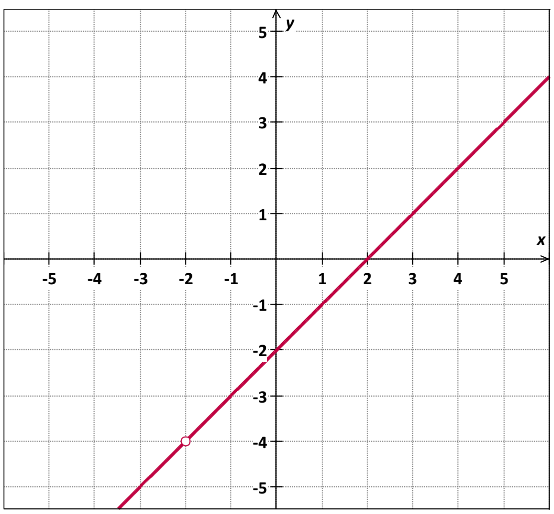
b. $f(x) = \dfrac{|2x|}{x} \quad at \quad x = 0$



c. $f(x) = \dfrac{x^2 - 4}{x + 2} \quad at \quad x = -2$



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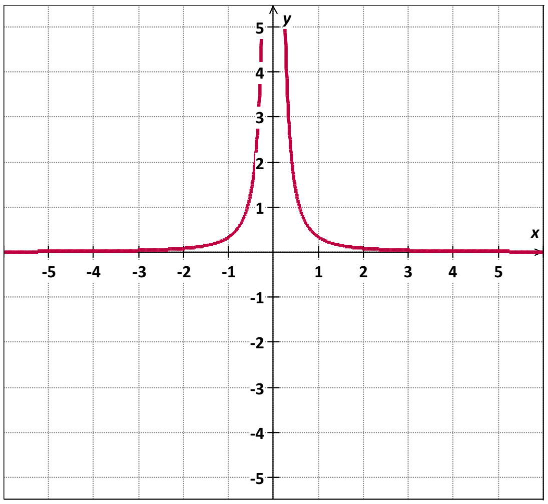
d. $f(x) = \frac{1}{3x^2}$ $\quad at \quad x = 0$



Intermediate Value Theorem

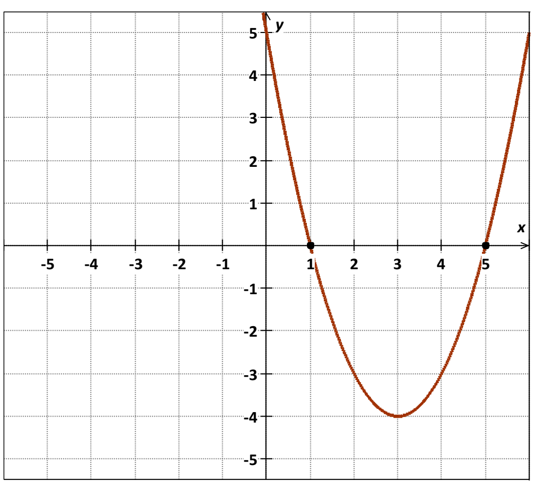
If $f(x)$is a continuous function and $a < b$ and there is a value n such that n is between $f(a)$ and $f(b)$, then there is a number$c$, such that $a < c < b$ and $f(c) = n$

The Location Principle

If $f(x)$ is a continuous function and $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value c, such that $a < c < b$ and $f(c) = 0$. That is, there is a zero between a and b.

Sample Problem 2: **Determine between which consecutive integers the real zeros of function are located on the given interval.**

a. $f(x) = (x - 3)^2 - 4$ $\qquad [0, 6]$



Continuity, End Behavior, and Limits Guided Notes

End Behavior

The end behavior of a function describes what the y -values do as $|x|$ becomes greater and greater.

When x becomes greater and greater, we say that x approaches infinity, and we write $x \to +\infty$.

When x becomes more and more negative, we say that x approaches negative infinity, and we write $x \to -\infty$.

The same notation can also be used with y or $f(x)$ and with real numbers instead of infinity.

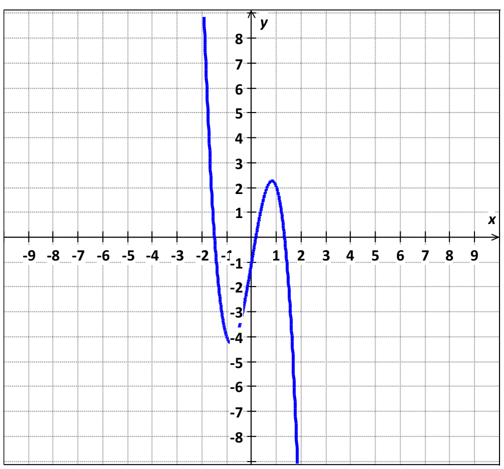
Left - End Behavior (as x becomes more and more negative): $\lim_{x \to -\infty} f(x)$

Right - End Behavior (as x becomes more and more positive): $\lim_{x \to +\infty} f(x)$

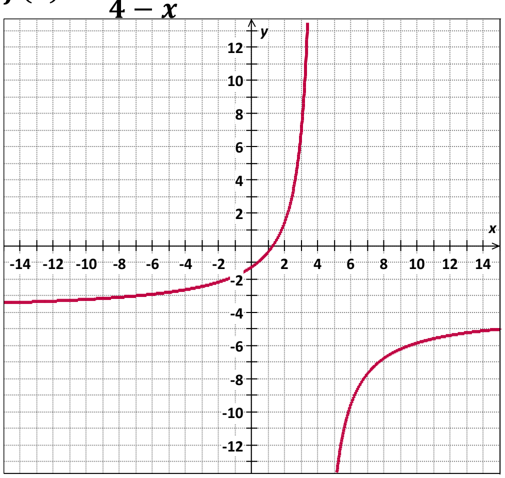
The $f(x)$ values may approach negative infinity, positive infinity, or a specific value.

Sample Problem 3: **Use the graph of each function to describe its end behavior. Support the conjecture numerically.**

a. $f(x) = -3x^3 + 6x - 1$



b. $f(x) = \dfrac{4x - 5}{4 - x}$



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Continuity, End Behavior, and Limits Guided Notes

Increasing, Decreasing, and Constant Functions

A function $\boldsymbol{f}$ is increasing on an interval $\boldsymbol{I}$ if and only if for every $\boldsymbol{a}$ and $\boldsymbol{b}$ contained in $\boldsymbol{I}$, $(\boldsymbol{a}) < \boldsymbol{f}(\boldsymbol{b})$, whenever $\boldsymbol{a} < \boldsymbol{b}$.

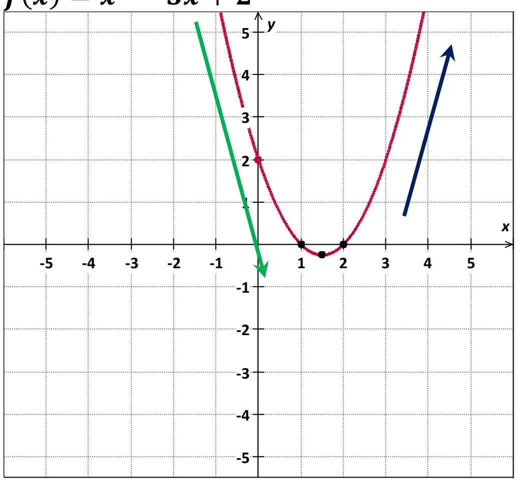
A function $\boldsymbol{f}$ is decreasing on an interval $\boldsymbol{I}$ if and only if for every $\boldsymbol{a}$ and $\boldsymbol{b}$ contained in $\boldsymbol{I}$, $\boldsymbol{f}(\boldsymbol{a}) > \boldsymbol{f}(\boldsymbol{b})$ whenever $\boldsymbol{a} < \boldsymbol{b}$.

A function $\boldsymbol{f}$ remains constant on an interval $\boldsymbol{I}$ if and only if for every $\boldsymbol{a}$ and $\boldsymbol{b}$ contained in $\boldsymbol{I}$, $\boldsymbol{f}(\boldsymbol{a}) = \boldsymbol{f}(\boldsymbol{b})$ whenever $\boldsymbol{a} < \boldsymbol{b}$.

Points in the domain of a function where the function changes from increasing to decreasing or from decreasing to increasing are called **critical points.**

Sample Problem 4: **Determine the interval(s) on which the function is increasing and the interval(s) on which the function is decreasing.**

a. $f(x) = x^2 - 3x + 2$



b. $f(x) = x^3 - \frac{1}{2}x^2 - 10x + 2$