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Function Operations and Composition of Functions

Unit 1 Lesson 6

Students will be able to:

Combine standard function types using arithmetic operations

Compose functions

Key Vocabulary:

Function operation

Composition of function

Decomposition of Composite Functions

Domain of composite function

Function Operations

Let f and g be any two functions. You can add, subtract, multiply or divide $f(x)$ and $g(x)$ to form a new function.

The domain of new function consist of the x -values that are in the domains of both $f(x)$ and $g(x)$.

When new function involves division, the domain does not include x -values for which the denominator is equal to zero.

Function Operations and Composition of Functions

Operation	*Definition*
Addition	$(f+g)(x)=f(x)+g(x)$
Subtraction	$(f-g)(x)=f(x)-g(x)$
Multiplication	$(f*g)(x)=f(x)*g(x)$
Division	$(f\div g)(x)=f(x)\div g(x)$ $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}\quad where\ g(x)\neq 0$

Sample Problem 1: : Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

a. $f(x) = x^2 + 2x - 1$ $\quad\quad$ $g(x) = x - 5$

Sample Problem 1: : Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

a. $f(x) = x^2 + 2x - 1$ $\qquad g(x) = x - 5$

$$(f+g)(x) = (x^2 + 2x - 1) + (x - 5)$$

$$(f+g)(x) = x^2 + 3x - 6$$

$$D_{f+g} = (-\infty, \infty)$$

Sample Problem 1: : Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

a. $f(x) = x^2 + 2x - 1$ $\qquad g(x) = x - 5$

$$(f-g)(x) = (x^2+2x-1) - (x-5)$$

$$(f-g)(x) = x^2 + 2x - 1 - x + 5$$

$$(f-g)(x) = x^2 + x + 4$$

$$D_{f-g} = (-\infty, \infty)$$

Sample Problem 1: : Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

a. $f(x) = x^2 + 2x - 1$ $\qquad g(x) = x - 5$

$$(f*g)(x) = (x^2+2x-1)*(x-5)$$

$$(f*g)(x) = x^3 - 3x^2 - 11x + 5$$

$$D_{f*g} = (-\infty, \infty)$$

Sample Problem 1: : Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

a. $f(x) = x^2 + 2x - 1$ $\qquad g(x) = x - 5$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2 + 2x - 1}{x - 5}$$

$$D_{\frac{f}{g}} = (-\infty, 5) \cup (5, \infty)$$

Sample Problem 1: : Find $(f+g)(x)$, $(f-g)(x)$, $(f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

b. $f(x) = x^2 - 81$ $\quad\quad$ $g(x) = x + 9$

Function Operations and Composition of Functions

Sample Problem 1: : **Find** $(f+g)(x), (f-g)(x), (f*g)(x)$, **and** $\left(\frac{f}{g}\right)(x)$ **for each** $f(x)$ **and** $g(x)$. **Determine the domain of each new function.**

b. $f(x) = x^2 - 81$ $\qquad g(x) = x + 9$

$$(f+g)(x) = (x^2 - 81) + (x + 9)$$

$$(f+g)(x) = x^2 + x - 72$$

$$D_{f+g} = (-\infty, \infty)$$

Function Operations and Composition of Functions

Sample Problem 1: **: Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.**

b. $f(x) = x^2 - 81$ $\quad\quad g(x) = x + 9$

$$(f-g)(x) = (x^2-81) - (x+9)$$

$$(f-g)(x) = x^2 - 81 - x - 9$$

$$(f-g)(x) = x^2 - x - 90$$

$$D_{f-g} = (-\infty, \infty)$$

Function Operations and Composition of Functions

Sample Problem 1: : Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

b. $f(x) = x^2 - 81$ $\qquad g(x) = x + 9$

$$(f*g)(x) = (x^2 - 81)*(x+9)$$

$$(f*g)(x) = x^3 + 9x^2 - 81x - 729$$

$$D_{f*g} = (-\infty, \infty)$$

Sample Problem 1: **: Find $(f+g)(x), (f-g)(x), (f*g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.**

b. $f(x) = x^2 - 81$ $\qquad g(x) = x + 9$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 81}{x+9} = \frac{(x+9)(x-9)}{x+9}$$

$$\left(\frac{f}{g}\right) = x - 9$$

$$D_{\frac{f}{g}} = (-\infty, -9) \cup (-9, \infty)$$

If the function can be simplified, determine the domain before simplifying!

Composition of Functions

The composition of function f with function g is defined by $(f \circ g)(x) = f(g(x))$

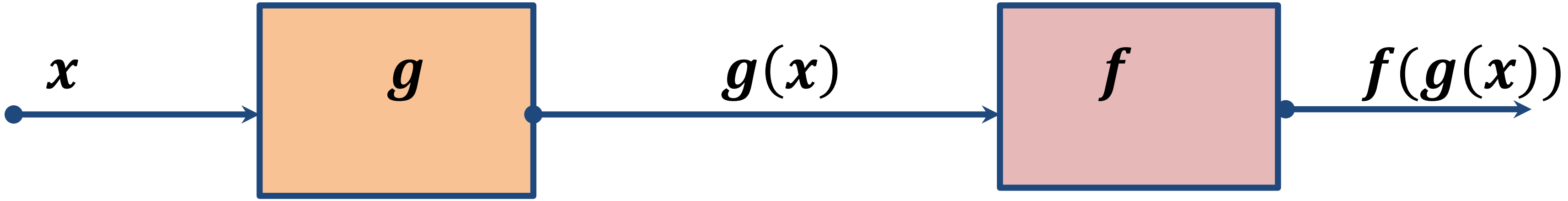
The domain of the composite function $f \circ g$ is the set of all such that:

1. x is in the domain of g and

2. $g(x)$ is in the domain of f.

Function Operations and Composition of Functions

x must be in the domain of g



$g(x)$ must be in the domain of f

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

a. $f(x) = 2x - 3$ $\qquad$ $g(x) = x + 1$

$(f \circ g)(x) = ?$ $\qquad$ $D_{f \circ g} = ?$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

a. $f(x) = 2x - 3$ $\qquad g(x) = x + 1$

$(f \circ g)(x) = ?$ $\qquad D_{f \circ g} = ?$

$$(f \circ g)(x) = f\big(g(x)\big)$$

$$f\big(g(x)\big) = 2\big(g(x)\big) - 3$$

$$f\big(g(x)\big) = 2(x + 1) - 3$$

$$D_{f \circ g} = (-\infty, \infty)$$

$$f\big(g(x)\big) = 2x + 2 - 3$$

$$f\big(g(x)\big) = 2x - 1$$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

b. $f(x) = x - 3$ $g(x) = x^2 + 1$

$(g \circ f)(x) = ?$ $D_{g \circ f} = ?$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

b. $f(x) = x - 3$ $\quad$ $g(x) = x^2 + 1$

$(g \circ f)(x) = ?$ $\quad$ $D_{g \circ f} = ?$

$$(g \circ f)(x) = g(f(x))$$

$$g(f(x)) = (f(x))^2 + 1$$

$$g(f(x)) = (x - 3)^2 + 1$$

$$g(f(x)) = x^2 - 6x + 9 + 1$$

$$g(f(x)) = x^2 - 6x + 10$$

$$D_{g \circ f} = (-\infty, \infty)$$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

c. $f(x) = \frac{2}{x-3}$ $\quad g(x) = \frac{1}{x}$ $\quad (f \circ g)(x) = ?$ $\quad D_{f \circ g} = ?$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

c. $f(x) = \frac{2}{x-3}$ $g(x) = \frac{1}{x}$ $(f \circ g)(x) = ?$

$$(f \circ g)(x) = f\big(g(x)\big)$$

$$f\big(g(x)\big) = \frac{2}{g(x) - 3}$$ $\frac{1}{x} \neq 0$ $x \neq 0$

$$f\big(g(x)\big) = \frac{2}{\frac{1}{x} - 3}$$ $\frac{1}{x} - 3 \neq 0$ $x \neq \frac{1}{3}$

$$f\big(g(x)\big) = \frac{2x}{1 - 3x}$$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

c. $f(x) = \frac{2}{x-3}$ $g(x) = \frac{1}{x}$ $D_{f \circ g} = ?$

$$D_g = (-\infty, 0) \cup (0, \infty)$$

$$D_{f \circ g} = (-\infty, 0) \cup (0, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

d. $f(x) = \frac{2}{x}$ $g(x) = \frac{1}{x}$ $(g \circ f)(x) = ?$ $D_{g \circ f} = ?$

Sample Problem 2: Find each composite function. Determine the domain of each composite function.

d. $f(x) = \frac{2}{x}$ $g(x) = \frac{1}{x}$ $(g \circ f)(x) = ?$

$$(g \circ f)(x) = g(f(x))$$

$$g(f(x)) = \frac{1}{f(x)}$$ $\frac{2}{x} \neq 0$ $x \neq 0$

$$g(f(x)) = \frac{1}{\frac{2}{x}}$$

$$g(f(x)) = \frac{x}{2}$$

Sample Problem 2: **Find each composite function. Determine the domain of each composite function.**

d. $f(x) = \frac{2}{x}$ $g(x) = \frac{1}{x}$ $D_{g \circ f} = ?$

$$D_g = (-\infty, 0) \cup (0, \infty)$$

$$D_{g \circ f} = (-\infty, 0) \cup (0, \infty)$$

Sample Problem 3: : Find and then evaluate each composite function.

a. $f(x) = \sqrt{x}$ $g(x) = x - 2$ $(f \circ g)(6) = ?$

Sample Problem 3: : Find and then evaluate each composite function.

a. $f(x) = \sqrt{x}$ $\quad g(x) = x - 2$ $\quad (f \circ g)(6) = ?$

$(f \circ g)(x) = f(g(x))$

$f(g(x)) = \sqrt{g(x)}$

$f(g(x)) = \sqrt{x - 2}$

$f(g(6)) = \sqrt{6 - 2}$

$f(g(6)) = \sqrt{4}$

$f(g(6)) = 2$

Sample Problem 3: : Find and then evaluate each composite function.

b. $f(x) = 6x - 1$ $\quad g(x) = \dfrac{x+3}{2}$ $\quad (g \circ f)(2) = ?$

Sample Problem 3: : Find and then evaluate each composite function.

b. $f(x) = 6x - 1$ $\quad g(x) = \dfrac{x+3}{2}$ $\quad (g \circ f)(2) = ?$

$$(g \circ f)(x) = g(f(x))$$

$$g(f(x)) = \frac{f(x) + 3}{2}$$

$$g(f(x)) = \frac{(6x - 1)\ + 3}{2}$$

$$g(f(x)) = \frac{6x + 2}{2} = \frac{2(3x + 1)}{2}$$

$$g(f(x)) = 3x + 1$$

$$g(f(2)) = 3 * 2 + 1$$

$$g(f(2)) = 7$$

Decomposition of Composite Functions

When you form a composite function, you "compose" two functions to form a new function. It is also possible to reverse this process. You can "decompose" a given function and express it as a composition of two functions. Although there is more than one way to do this, there is often a "natural" selection that comes to mind first.

Sample Problem 4: **Express $h(x)$ as a composition of two functions f and g $(f \circ g)(x)$.**

a. $h(x) = (x^3 - 3x)^2$

Sample Problem 4: **Express $h(x)$ as a composition of two functions f and g $(f \circ g)(x)$.**

a. $h(x) = (x^3 - 3x)^2$

$h(x) = (f \circ g)(x) = f(g(x))$

$f(g(x)) = (g(x))^2 = (x^3 - 3x)^2$

$f(x) = x^2$ $\qquad$ $g(x) = x^3 - 3x$

Sample Problem 4: Express $h(x)$ as a composition of two functions f and g $(f \circ g)(x)$.

b. $h(x) = \dfrac{3}{3x-5}$

Sample Problem 4: **Express $h(x)$ as a composition of two functions f and g $(f \circ g)(x)$.**

b. $h(x) = \dfrac{3}{3x - 5}$

$$h(x) = (f \circ g)(x) = f\big(g(x)\big)$$

$$f\big(g(x)\big) = \frac{3}{g(x) - 5} = \frac{3}{3x - 5}$$

$$f(x) = \frac{3}{x - 5} \qquad g(x) = 3x$$