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Function Operations and Composition of Functions Guided Notes

Function Operations

Let f and g be any two functions. You can add, subtract, multiply or divide $f(x)$ and $g(x)$ to form a new function.

The domain of new function consist of the x -values that are in the domains of both $f(x)$ and $g(x)$**.** When new function involves division, the domain does not include x -values for which the denominator is equal to zero.

Operation	***Definition***
Addition	$(f+g)(x)=f(x)+g(x)$
Subtraction	$(f-g)(x)=f(x)-g(x)$
Multiplication	$(f*g)(x)=f(x)*g(x)$
Division	$(f\div g)(x)=f(x)\div g(x)$ $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}\quad where\ g(x)\neq 0$

Sample Problem 1: Find $(f+g)(x), (f-g)(x), (f*g)(x), and\ \left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

a. $f(x)=x^2+2x-1$
$g(x)=x-5$

$(f+g)(x)=(x^2+2x-1)+(x-5)$
$(f+g)(x)=x^2+3x-6$

$D_{f+g}=(-\infty,\infty)$

$(f-g)(x)=(x^2+2x-1)-(x-5)$
$(f-g)(x)=x^2+2x-1-x+5$
$(f-g)(x)=x^2+x+4$

$D_{f-g}=(-\infty,\infty)$

$(f*g)(x)=(x^2+2x-1)*(x-5)$
$(f*g)(x)=x^3-3x^2-11x+5$

$D_{f*g}=(-\infty,\infty)$

$$\left(\frac{f}{g}\right)(x)=\frac{x^2+2x-1}{x-5}$$

$D_{\frac{f}{g}}=(-\infty,5)\cup(5,\infty)$

b. $f(x)=x^2-81$
$g(x)=x+9$

$(f+g)(x)=(x^2-81)+(x+9)$
$(f+g)(x)=x^2+x-72$

$D_{f+g}=(-\infty,\infty)$

$(f-g)(x)=(x^2-81)-(x+9)$
$(f-g)(x)=x^2-81-x-9$
$(f-g)(x)=x^2-x-90$

$D_{f-g}=(-\infty,\infty)$

$(f*g)(x)=(x^2-81)*(x+9)$
$(f*g)(x)=x^3+9x^2-81x-729$

$D_{f*g}=(-\infty,\infty)$

$$\left(\frac{f}{g}\right)(x)=\frac{x^2-81}{x+9}=\frac{(x+9)(x-9)}{x+9}=x-9$$

$D_{\frac{f}{g}}=(-\infty,-9)\cup(-9,\infty)$

If the function can be simplified, determine the domain before simplifying!

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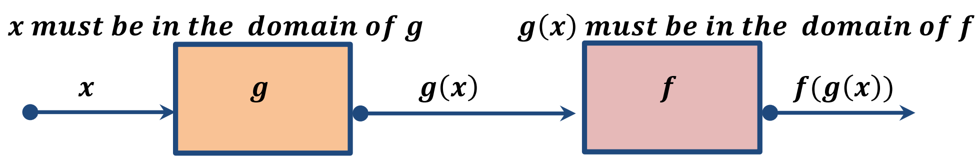
Composition of Functions

The composition of function f with function g is defined by $(f \circ g)(x) = f(g(x))$

The domain of the composite function $f \circ g$ is the set of all such that:

1. x is in the domain of g and

2. $g(x)$ is in the domain of f.



Sample Problem 2: **Find each composite function. Determine the domain of each composite function.**

a. $f(x) = 2x - 3 \qquad g(x) = x + 1$

$(f \circ g)(x) = ? \qquad D_{f \circ g} = ?$

$(f \circ g)(x) = f(g(x))$
$f(g(x)) = 2(g(x)) - 3$
$f(g(x)) = 2(x+1) - 3$
$f(g(x)) = 2x + 2 - 3$
$f(g(x)) = 2x - 1$

$D_{f \circ g} = (-\infty, \infty)$

b. $f(x) = x - 3 \qquad g(x) = x^2 + 1$

$(g \circ f)(x) = ? \qquad D_{g \circ f} = ?$

$(g \circ f)(x) = g(f(x))$
$g(f(x)) = (f(x))^2 + 1$
$g(f(x)) = (x-3)^2 + 1$
$g(f(x)) = x^2 - 6x + 9 + 1$
$g(f(x)) = x^2 - 6x + 10$

$D_{g \circ f} = (-\infty, \infty)$

c. $f(x) = \frac{2}{x-3} \qquad g(x) = \frac{1}{x}$

$(f \circ g)(x) = ? \quad D_{f \circ g} = ?$

$(f \circ g)(x) = f(g(x))$

$f(g(x)) = \frac{2}{g(x) - 3} \qquad \frac{1}{x} \neq 0 \qquad x \neq 0$

$f(g(x)) = \frac{2}{\frac{1}{x} - 3} \qquad \frac{1}{x} - 3 \neq 0 \qquad x \neq \frac{1}{3}$

$f(g(x)) = \frac{2x}{1 - 3x}$

$D_g = (-\infty, 0) \cup (0, \infty)$

$D_{f \circ g} = (-\infty, 0) \cup (0, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

d. $f(x) = \frac{2}{x} \qquad g(x) = \frac{1}{x}$

$(g \circ f)(x) = ? \quad D_{g \circ f} = ?$

$(g \circ f)(x) = g(f(x))$

$g(f(x)) = \frac{1}{f(x)} \qquad \frac{2}{x} \neq 0 \qquad x \neq 0$

$g(f(x)) = \frac{1}{\frac{2}{x}}$

$g(f(x)) = \frac{x}{2}$

$D_g = (-\infty, 0) \cup (0, \infty)$

$D_{g \circ f} = (-\infty, 0) \cup (0, \infty)$

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Sample Problem 3: **Find and then evaluate each composite function.**

a. $f(x) = \sqrt{x} \quad g(x) = x - 2$

$(f \circ g)(6) = ?$

$(f \circ g)(x) = f(g(x))$

$f(g(x)) = \sqrt{g(x)}$

$f(g(x)) = \sqrt{x-2}$

$f(g(6)) = \sqrt{6-2}$

$f(g(6)) = \sqrt{4}$

$f(g(6)) = 2$

b. $f(x) = 6x - 1 \quad g(x) = \frac{x+3}{2}$

$(g \circ f)(2) = ?$

$(g \circ f)(x) = g(f(x))$

$g(f(x)) = \frac{f(x)+3}{2}$

$g(f(x)) = \frac{(6x-1)+3}{2}$

$g(f(x)) = \frac{6x+2}{2} = \frac{2(3x+1)}{2}$

$g(f(x)) = 3x+1$

$g(f(2)) = 3 * 2 + 1$

$g(f(2)) = 7$

Decomposition of Composite Functions

When you form a composite function, you "compose" two functions to form a new function. It is also possible to reverse this process. You can "decompose" a given function and express it as a composition of two functions. Although there is more than one way to do this, there is often a "natural" selection that comes to mind first.

Sample Problem 4: **Express $h(x)$ as a composition of two functions f and g $(f \circ g)(x)$.**

a. $h(x) = (x^3 - 3x)^2$

$h(x) = (x^3 - 3x)^2$

$h(x) = (f \circ g)(x) = f(g(x))$

$f(g(x)) = (g(x))^2 = (x^3 - 3x)^2$

$f(x) = x^2$

$g(x) = x^3 - 3x$

b. $h(x) = \frac{3}{3x-5}$

$h(x) = \frac{3}{3x-5}$

$h(x) = (f \circ g)(x) = f(g(x))$

$f(g(x)) = \frac{3}{g(x)-5} = \frac{3}{3x-5}$

$f(x) = \frac{3}{x-5}$

$g(x) = 3x$