

Function Operations and Composition of Functions

Guided Notes

Function Operations

Let f and g be any two functions. You can add, subtract, multiply or divide $f(x)$ and $g(x)$ to form a new function.

The domain of new function consist of the x -values that are in the domains of both $f(x)$ and $g(x)$. When new function involves division, the domain does not include x -values for which the denominator is equal to zero.

| <i>Operation</i> | <i>Definition</i> |
|-----------------------|---|
| <i>Addition</i> | $(f + g)(x) = f(x) + g(x)$ |
| <i>Subtraction</i> | $(f - g)(x) = f(x) - g(x)$ |
| <i>Multiplication</i> | $(f * g)(x) = f(x) * g(x)$ |
| <i>Division</i> | $(f \div g)(x) = f(x) \div g(x)$ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$ |

Sample Problem 1: Find $(f + g)(x)$, $(f - g)(x)$, $(f * g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Determine the domain of each new function.

a. $f(x) = x^2 + 2x - 1$
 $g(x) = x - 5$

b. $f(x) = x^2 - 81$
 $g(x) = x + 9$

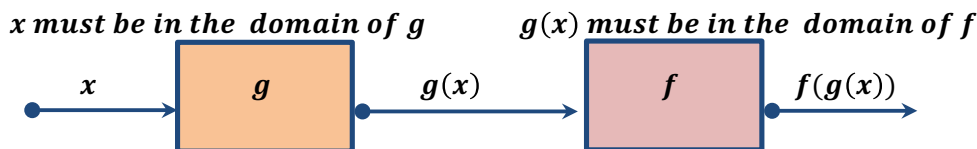
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Composition of Functions

The composition of function f with function g is defined by $(f \circ g)(x) = f(g(x))$

The domain of the composite function $f \circ g$ is the set of all such that:

1. x is in the domain of g and
2. $g(x)$ is in the domain of f .



Sample Problem 2: Find each composite function. Determine the domain of each composite function.

a. $f(x) = 2x - 3$ $g(x) = x + 1$
 $(f \circ g)(x) = ?$ $D_{f \circ g} = ?$

b. $f(x) = x - 3$ $g(x) = x^2 + 1$
 $(g \circ f)(x) = ?$ $D_{g \circ f} = ?$

c. $f(x) = \frac{2}{x-3}$ $g(x) = \frac{1}{x}$
 $(f \circ g)(x) = ?$ $D_{f \circ g} = ?$

d. $f(x) = \frac{2}{x}$ $g(x) = \frac{1}{x}$
 $(g \circ f)(x) = ?$ $D_{g \circ f} = ?$

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Sample Problem 3: Find and then evaluate each composite function.

a. $f(x) = \sqrt{x}$ $g(x) = x - 2$
 $(f \circ g)(6) = ?$

b. $f(x) = 6x - 1$ $g(x) = \frac{x + 3}{2}$
 $(g \circ f)(2) = ?$

Decomposition of Composite Functions

When you form a composite function, you “compose” two functions to form a new function. It is also possible to reverse this process. You can “decompose” a given function and express it as a composition of two functions. Although there is more than one way to do this, there is often a “natural” selection that comes to mind first.

Sample Problem 4: Express $h(x)$ as a composition of two functions f and g ($f \circ g$)(x).

a. $h(x) = (x^3 - 3x)^2$

b. $h(x) = \frac{3}{3x - 5}$