***Function Operations***

Let $f$and$g$be any two functions. You can add, subtract, multiply or divide $f\left(x\right)$and$g(x)$to form a new function.

The domain of new function consist of the $x$ -values that are in the domains of both $f\left(x\right)$ and$g(x)$**.** When new function

involves division, the domain does not include$ x$ -values for which the denominator is equal to zero.

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| ***Operation*** | ***Definition*** |
| ***Addition*** | $$\left(f+g\right)\left(x\right)=f\left(x\right)+g(x)$$ |
| ***Subtraction*** | $$\left(f-g\right)\left(x\right)=f\left(x\right)-g(x)$$ |
| ***Multiplication*** | $$\left(f\*g\right)\left(x\right)=f\left(x\right)\*g(x)$$ |
| ***Division*** | $$\left(f÷g\right)\left(x\right)=f\left(x\right)÷g(x)$$$$\left(\frac{f}{g}\right)\left(x\right)=\frac{f\left(x\right)}{g(x)} where g(x)\ne 0$$ |

**Sample Problem 1: Find** $\left(f+g\right)\left(x\right), \left(f-g\right)\left(x\right),\left(f\*g\right)\left(x\right), and $$\left(\frac{f}{g}\right)\left(x\right) $**for each** $f\left(x\right)$ **and** $g(x)$**. Determine the domain of each new function.**

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| **a.**  | $$f\left(x\right)=x^{2}+2x-1 $$$$g\left(x\right)=x-5$$ | **b.** | $$f\left(x\right)=x^{2}-81 $$$$g\left(x\right)=x+9$$ |
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**Composition of Functions**

The composition of function$ f$ with function $g $is defined by $\left(f∘g\right)\left(x\right)=f\left(g(x)\right)$

The domain of the composite function$ f∘g$ is the set of all such that:

1.$ x$ is in the domain of$ g$ and

2. $g\left(x\right) $is in the domain of $f$.

$x must be in the domain of g$$g\left(x\right) must be in the domain of f$

$ x g g\left(x\right) f f(g\left(x\right)) $

**Sample Problem 2: Find each composite function. Determine the domain of each composite function.**

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| **a.**  | $f\left(x\right)=2x-3 $$g\left(x\right)=x+1$$\left(f∘g\right)\left(x\right)=?$$D\_{f∘g}=?$ | **b.** | $f\left(x\right)=x-3 $$g\left(x\right)=x^{2}+1$$\left(g∘f\right)\left(x\right)=?$$D\_{g∘f}=?$ |
|  |   |  |  |
| **c.**  | $$f\left(x\right)=\frac{2}{x-3} g\left(x\right)=\frac{1}{x}$$$\left(f∘g\right)\left(x\right)=?$$D\_{f∘g}=?$ | **d.** | $$f\left(x\right)=\frac{2}{x} g\left(x\right)=\frac{1}{x}$$$\left(g∘f\right)\left(x\right)=?$$D\_{g∘f}=?$ |
|  |  |  |  |

**Sample Problem 3: Find and then evaluate each composite function.**

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| --- | --- | --- | --- |
| **a.**  | $$f\left(x\right)=\sqrt{x} g\left(x\right)=x-2$$$$\left(f∘g\right)\left(6\right)=?$$ | **b.** | $$f\left(x\right)=6x-1 g\left(x\right)=\frac{x+3}{2}$$$$\left(g∘f\right)\left(2\right)=?$$ |
|  |   |  |  |

**Decomposition of Composite Functions**

When you form a composite function, you “compose” two functions to form a new function. It is also possible to reverse this process. You can “decompose” a given function and express it as a composition of two functions. Although there is more than one way to do this, there is often a “natural” selection that comes to mind first.

**Sample Problem 4:**  **Express**$ h\left(x\right)$ **as a composition of two functions** $f$ **and** $g$$\left(f∘g\right)\left(x\right).$

|  |  |  |  |
| --- | --- | --- | --- |
| **a.**  | $$h\left(x\right)=\left(x^{3}-3x\right)^{2}$$ | **b.** | $$h\left(x\right)=\frac{3}{3x-5}$$ |
|  |  |  |  |