PreCalculusCoach.com

# Parent Functions and Transformations

Unit 1 Lesson 5

## Students will be able to:

Identify the effect on the graph of replacing $\boldsymbol{f(x)}$ by $\boldsymbol{f(x)+k}$, $\boldsymbol{kf(x)}$, $\boldsymbol{f(kx)}$, and $\boldsymbol{f(x+k)}$ for specific values of $\boldsymbol{k}$ (both positive and negative); find the value of k given the graphs.

## Key Vocabulary:

Parent function

Transformation

Translation

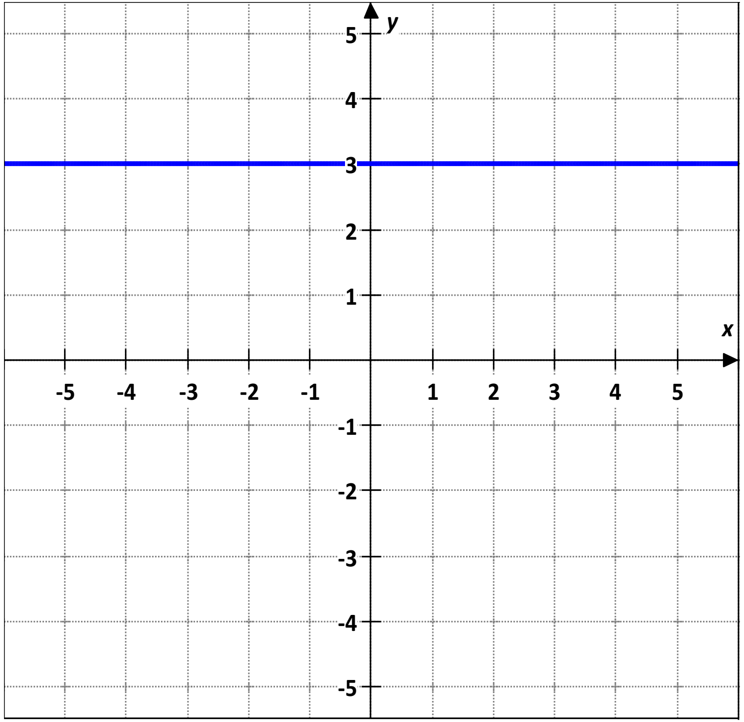
Dilation

**A family of functions** is a group of functions with graphs that display one or more similar characteristics.

**The Parent Function** is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

# Family - Constant Function

**Graph:**



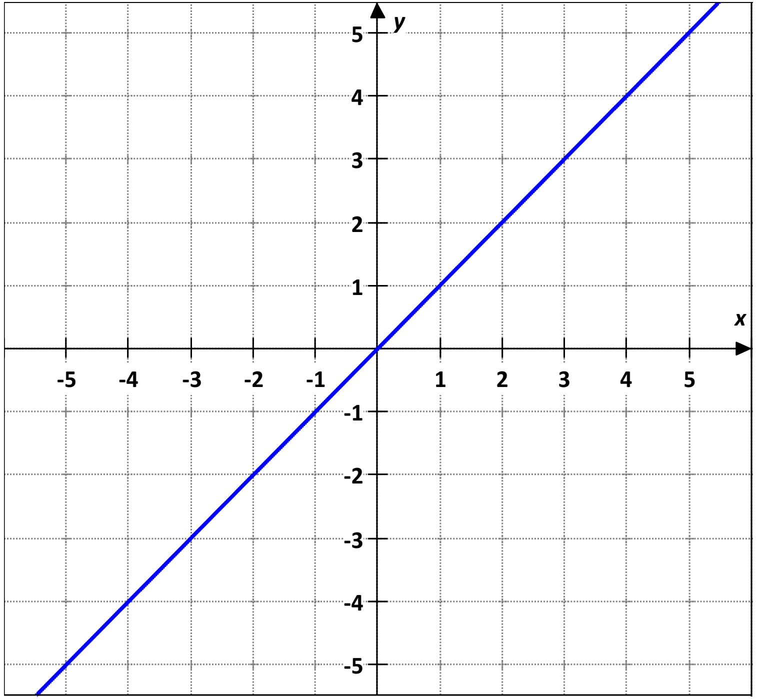

**Rule** $f(x) = c$

**Domain** $= (-\infty, \infty)$

**Range** $= [c]$

# Family - Linear Function

**Graph:**



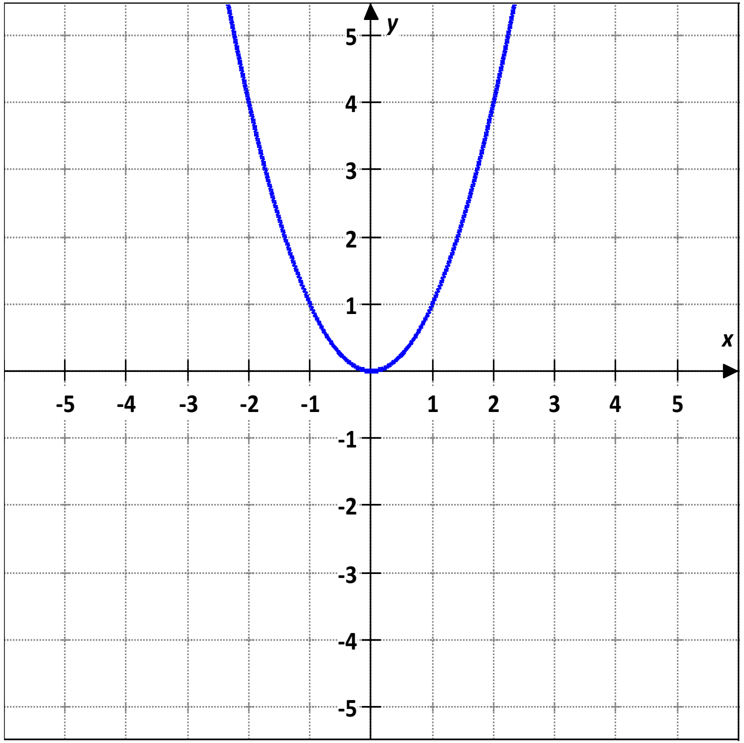

**Rule** $f(x) = x$

**Domain** $= (-\infty, \infty)$

**Range** $= (-\infty, \infty)$

# Family - Quadratic Function

**Graph:**



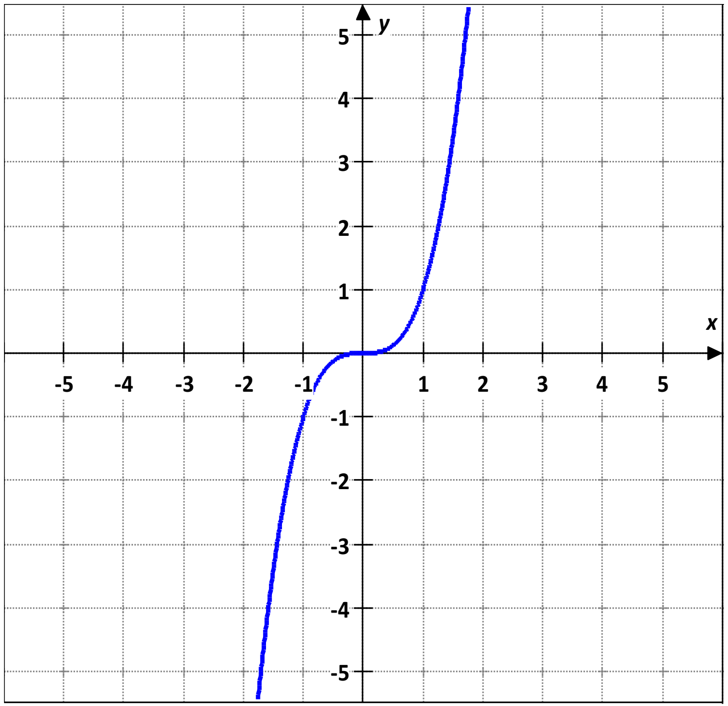

**Rule** $f(x) = x^2$

**Domain** $= (-\infty, \infty)$

**Range** $= [0, \infty)$

# Family - Cubic Function

**Graph:**



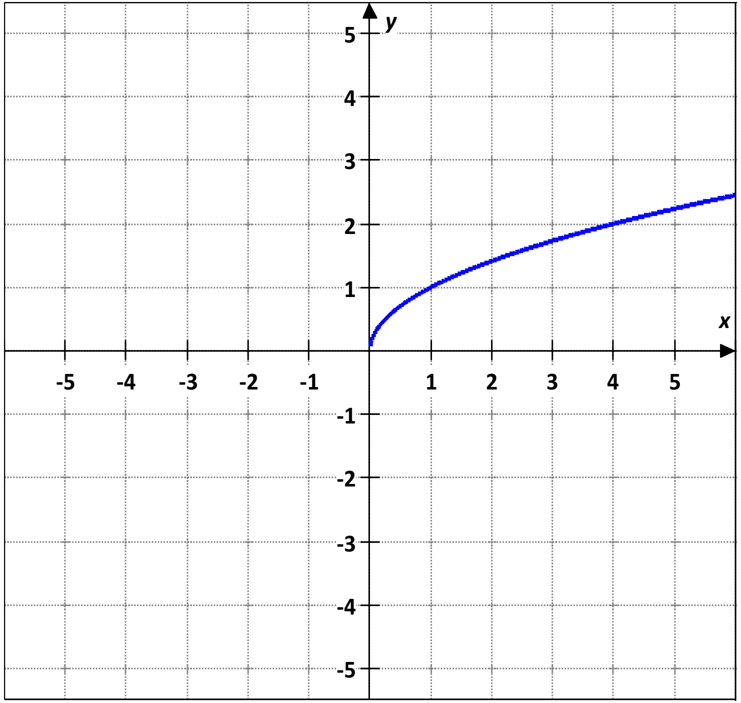

**Rule** $f(x) = x^3$

**Domain** $= (-\infty, \infty)$

**Range** $= (-\infty, \infty)$

# Family - Square Root Function

**Graph:**



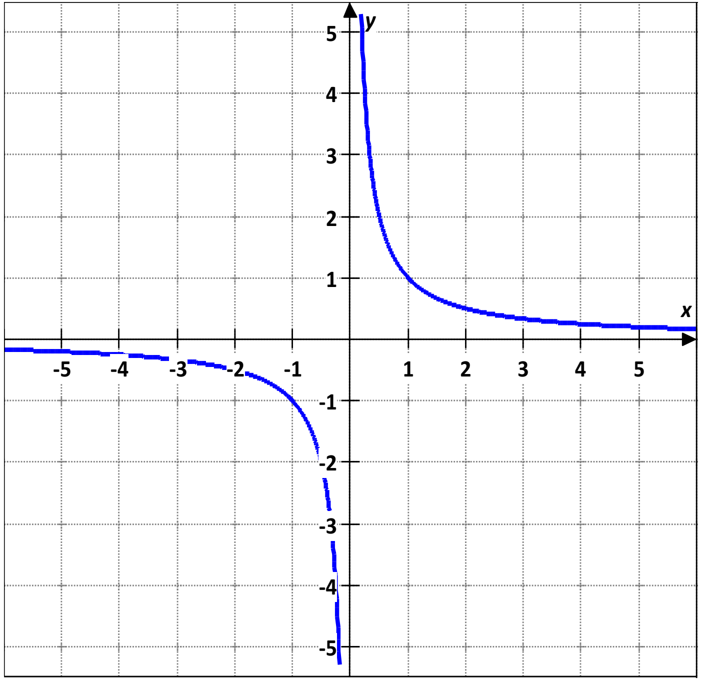

**Rule** $f(x) = \sqrt{x}$

**Domain** $= [0, \infty)$

**Range** $= [0, \infty)$

# Family - Reciprocal Function

**Graph:**



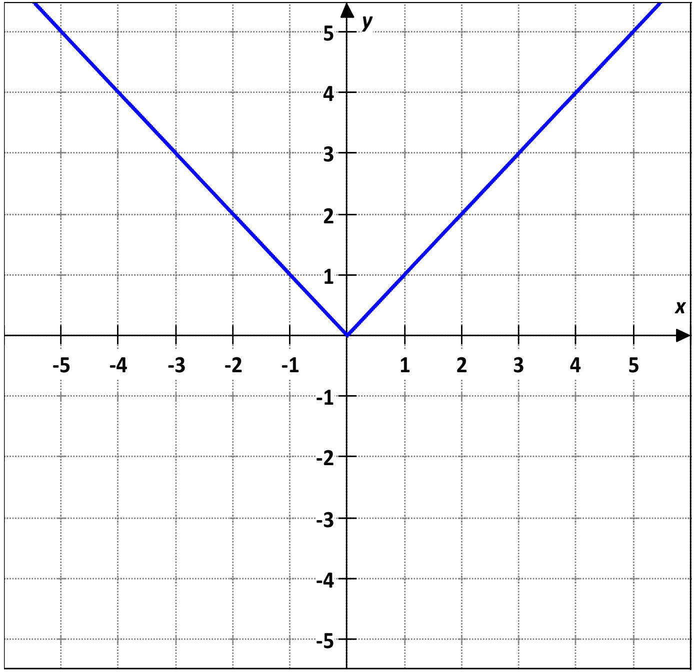

**Rule** $f(x) = \frac{1}{x}$

$D = (-\infty, 0) \cup (0, \infty)$

$R = (-\infty, 0) \cup (0, \infty)$

# Family - Absolut Value Function

Graph:




Rule $f(x) = |x|$

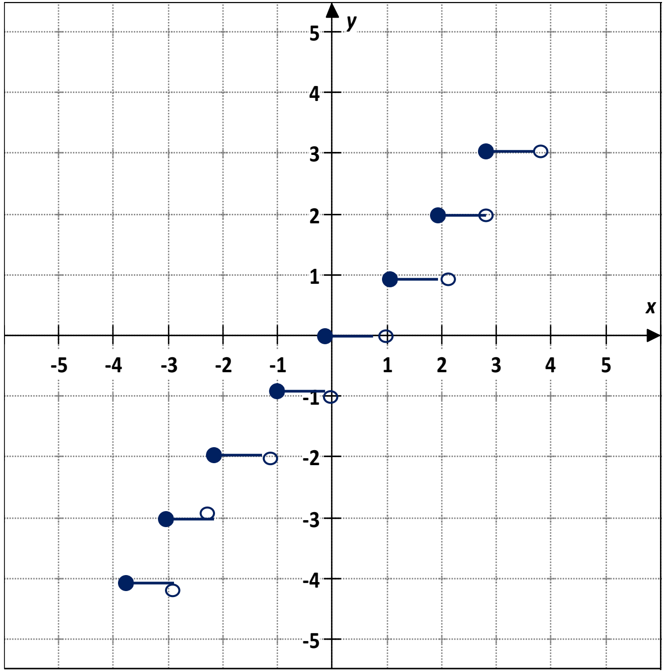
$$|x| = \begin{cases} -x & if \;\; x < 0 \\ x & if \;\; x \geq 0 \end{cases}$$

$D = (-\infty, \infty)$

$R = [0, \infty)$

# Family - Greatest Integer Function

Graph:




Rule $f(x) = [\![x]\!]$

$D = (-\infty, \infty)$

$R =$ All Integer

# *Transformations*

A change in the size or position of a figure or graph of the function is called a transformation.

***Rigid transformations*** change only the position of the graph, leaving the size and shape unchanged.

***Non rigid transformations*** distort the shape of the graph.

# *Rigid transformations*

## *Vertical Translations*

| Appearance in Function | Transformation of Graph | Transformation of Point |
|---|---|---|
| $f(x) \rightarrow f(x) + a$ | $a$ *units up* | $(x, y) \rightarrow (x, y + a)$ |
| $f(x) \rightarrow f(x) - a$ | $a$ *units down* | $(x, y) \rightarrow (x, y - a)$ |

## *Rigid transformations*

### *Horizontal Translations*

| Appearance in Function | Transformation of Graph | Transformation of Point |
|---|---|---|
| $f(x) \to f(x-b)$ | $b$ *units right* | $(x, y) \to (x+b, y)$ |
| $f(x) \to f(x+b)$ | $b$ *units left* | $(x, y) \to (x-b, y)$ |

## *Rigid transformations*

### *Reflections in x-axes*

| Appearance in Function | Transformation of Graph | Transformation of Point |
|---|---|---|
| $f(x) \rightarrow -f(x)$ | $reflected\ in\ the\ \boldsymbol{x}\ \boldsymbol{axis}$ | $(x, y) \rightarrow (x, -y)$ |

## Rigid transformations

### Reflections in y-axes

| Appearance in Function | Transformation of Graph | Transformation of Point |
|---|---|---|
| $f(x) \to f(-x)$ | $reflected\ in\ the\ \boldsymbol{y}\ \boldsymbol{axis}$ | $(x, y) \to (-x, y)$ |

# *Non rigid transformations*

## *Vertical Dilations*

| Appearance in Function | Transformation of Graph | Transformation of Point |
|---|---|---|
| $f(x) \to cf(x)$<br>$c > 1$ | *expanded vertically* | $(x, y) \to (cx, y)$ |
| $f(x) \to cf(x)$<br>$0 < c < 1$ | *compressed vertically* | $(x, y) \to (cx, y)$ |

# *Non rigid transformations*

## *Horizontal Dilations*

| Appearance in Function | Transformation of Graph | Transformation of Point |
|---|---|---|
| $f(x) \rightarrow f(dx)$<br>$d > 1$ | *compressed horizontally* | $(x, y) \rightarrow \left(\frac{x}{d}, y\right)$ |
| $f(x) \rightarrow f(dx)$<br>$0 < d < 1$ | *expanded horizontally* | $(x, y) \rightarrow \left(\frac{x}{d}, y\right)$ |

**Sample Problem 1: Identify the parent function and describe the transformations.**

a. $f(x) = (x-1)^2$

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a. $f(x) = (x - 1)^2$

**Parent :** $f(x) = x^2$

**Transformation:** Translation 1 unit right

**Sample Problem 1: Identify the parent function and describe the transformations.**

b. $f(x) = x^3 - 5$

**Sample Problem 1: Identify the parent function and describe the transformations.**

**b.** $f(x) = x^3 - 5$

**Parent :** $f(x) = x^3$

**Transformation:** Translation 5 units down

**Sample Problem 1: Identify the parent function and describe the transformations.**

c. $f(x) = -|x + 4|$

**Sample Problem 1:** **Identify the parent function and describe the transformations.**

c. $f(x) = -|x + 4|$

**Parent :** $f(x) = |x|$

**Transformation:** Reflection in x-axis
Translation 4 units left

**Sample Problem 1: Identify the parent function and describe the transformations.**

d. $f(x) = 3x^2 + 7$

**Sample Problem 1: Identify the parent function and describe the transformations.**

d. $f(x) = 3x^2 + 7$

**Parent :** $f(x) = x^2$

**Transformation:** Expand vertically by a factor of 3
Translation 7 units up

**Sample Problem 2:** **Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$ .**

a. **Quadratic - expanded horizontally by a factor of 2, translated 7 units up.**

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a. **Quadratic - expanded horizontally by a factor of 2, translated 7 units up.**

$$f(x) = \frac{1}{2}x^2 + 7$$

**Sample Problem 2:** **Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$ .**

b. **Cubic - reflected over the $x$ axis and translated 9 units down.**

**Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.**

**b. Cubic - reflected over the $x$ axis and translated 9 units down.**

$$f(x) = -x^3 - 9$$

**Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$ .**

c. **Absolute value - translated 3 units up, translated 8 units' right.**

**Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$ .**

c. **Absolute value - translated 3 units up, translated 8 units right.**

$$f(x) = |x - 8| + 3$$

**Sample Problem 2:** **Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$ .**

d. Reciprocal - translated 1 unit up.

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d. Reciprocal - translated 1 unit up.

$$f(x) = \frac{1}{x} + 1$$

**Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

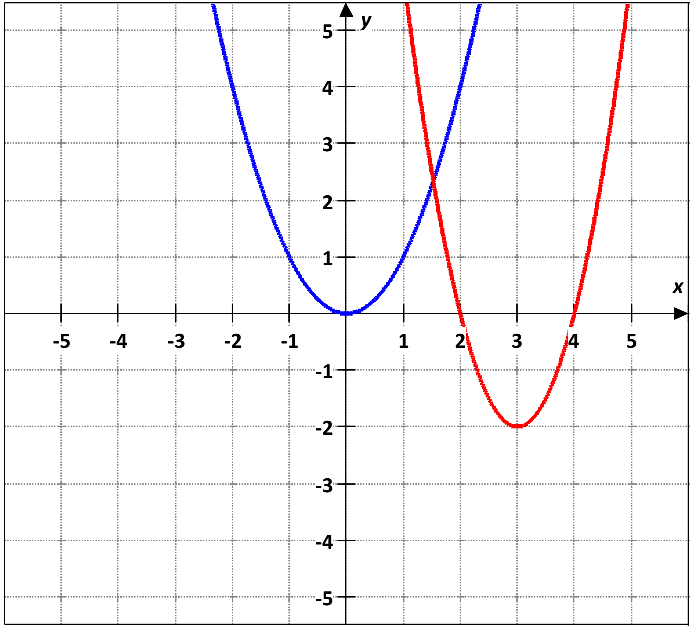
a. $h(x) = 2(x-3)^2 - 2$

**Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

a. $h(x) = 2(x-3)^2 - 2$

$h(x) = 2(x-3)^2 - 2$ →

**Parent function** $f(x) = x^2$ →




Transformation:

Compressed horizontally by a factor of 2

Translated 2 units down

Translated 3 units right

$D = (-\infty, \infty)$ $R = (-2, \infty)$

**Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

**b.** $h(x) = \sqrt{x-5} + 3$

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**b.** $h(x) = \sqrt{x-5} + 3$

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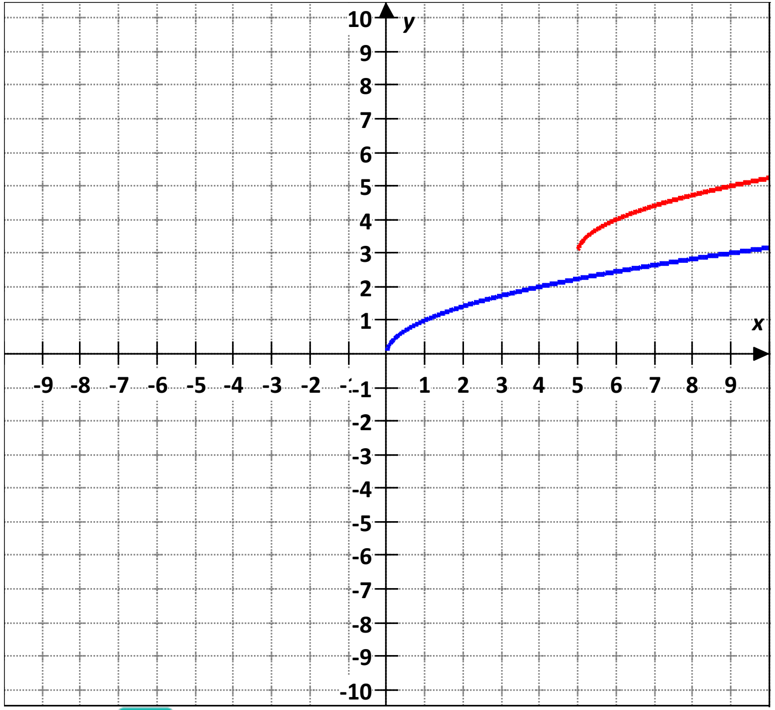
**Parent function** $f(x) = \sqrt{x}$




Transformation:

Translated 3 units up

Translated 5 units right

$D = [5. \infty)$ $R = (3, \infty)$

**Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

c. $h(x) = -|x + 4| - 1$

**Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

c. $h(x) = -|x+4| - 1$

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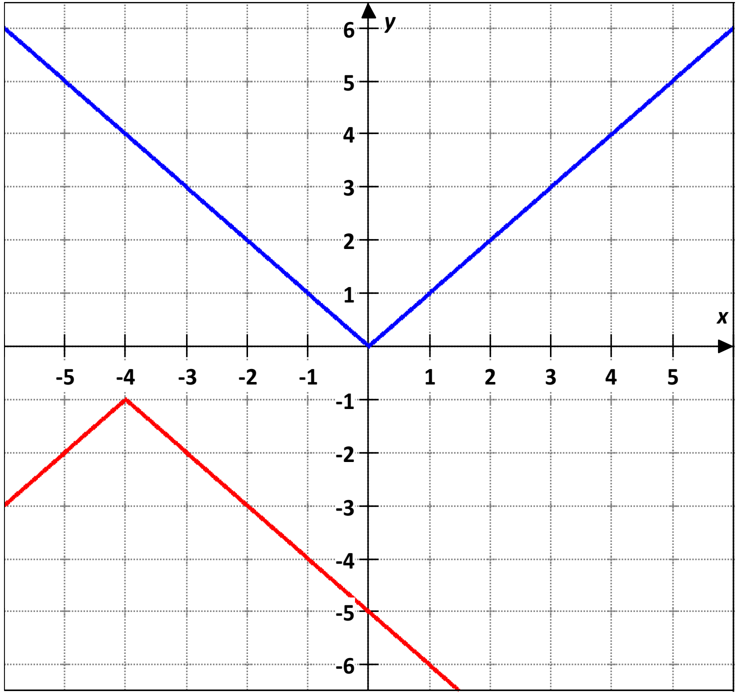
**Parent function** $f(x) = |x|$




Transformation:
Reflected in the x axis
Translated 1 unit down
Translated 4 units left

$D = (-\infty . \infty)$ $\quad R = (-\infty, -1]$

# Transformations with Absolute Value

$$h(x) = |f(x)|$$

This transformation reflects any portion of the graph of $f(x)$ that is below the $x$ -axis so that it is above the $x$ -axis.

# Transformations with Absolute Value

$$h(x) = f(|x|)$$

This transformation results, in the portion of the graph of $f(x)$ that is to the left of the $y$-axis, being replaced by a reflection of the portion to the right of the $y$ -axis.

## Sample Problem 4: Graph each function.

a. $f(x) = x^3 - 2x$ Graph $h(x) = |x^3 - 2x|$

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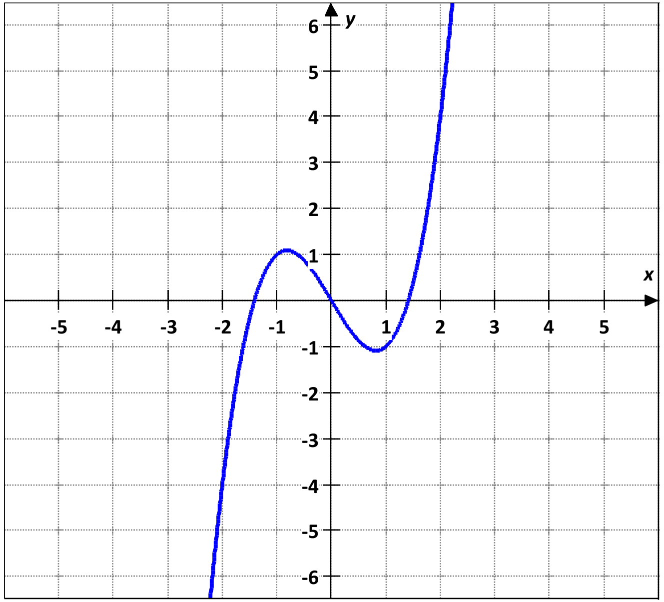
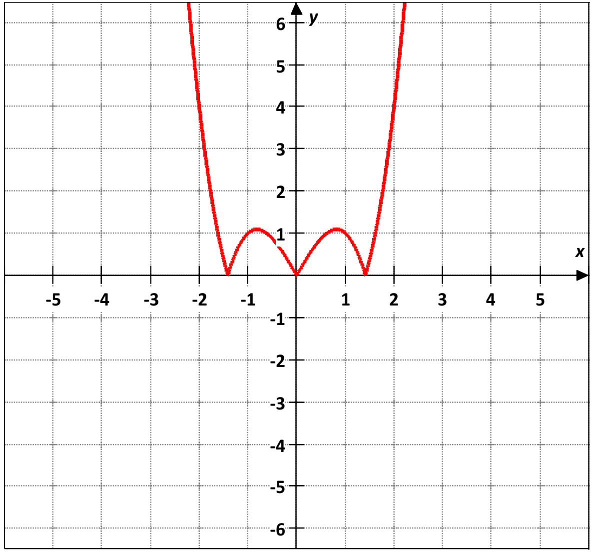
$f(x) = x^3 - 2x$ →



## Sample Problem 4: Graph each function.

a. $f(x) = x^3 - 2x$ Graph $h(x) = |x^3 - 2x|$

$h(x) = |x^3 - 2x|$ →



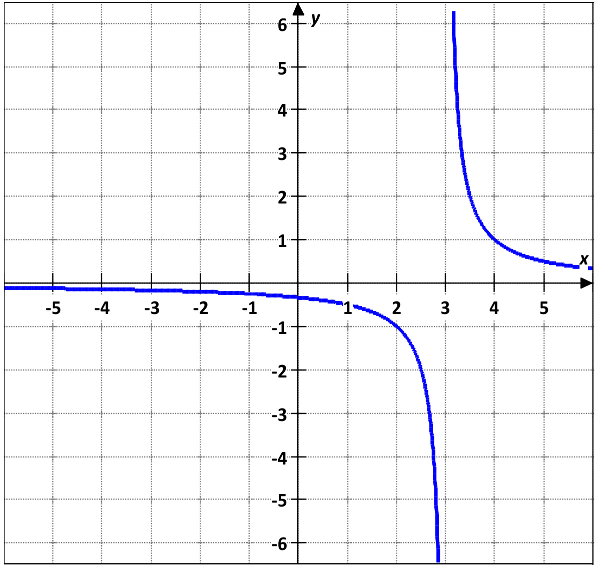
## Sample Problem 4: Graph each function.

**b.** $f(x) = \dfrac{1}{x-3}$ Graph $h(x) = \dfrac{1}{|x-3|}$

## Sample Problem 4: Graph each function.

**b.** $f(x) = \dfrac{1}{x-3}$ **Graph** $h(x) = \dfrac{1}{|x-3|}$

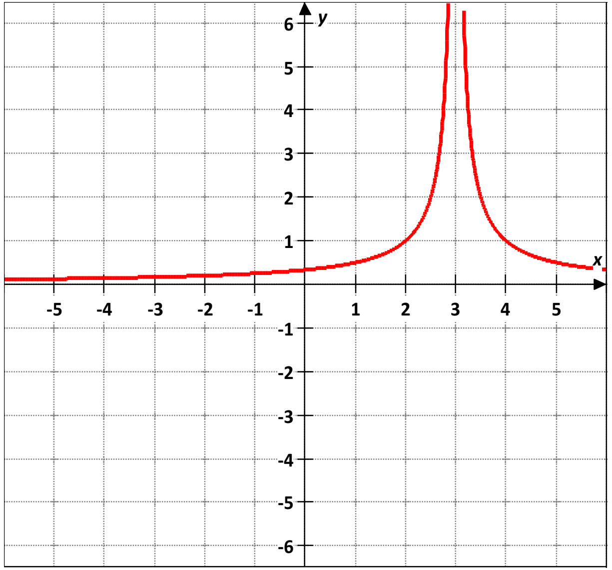
$$f(x) = \frac{1}{x-3}$$



## Sample Problem 4: Graph each function.

b. $f(x) = \dfrac{1}{x-3}$ Graph $h(x) = \dfrac{1}{|x-3|}$

$$h(x) = \frac{1}{|x-3|}$$



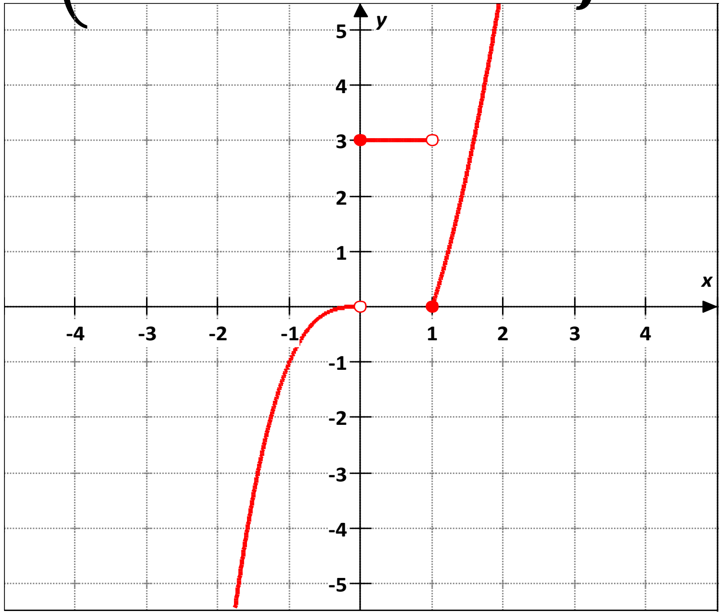
# Graph a Piecewise-Defined Function

## Sample Problem 5: Graph each piecewise function.

a. $$f(x) = \begin{cases} -x^3 & if\ x < 0 \\ 3 & if\ 0 \le x < 1 \\ 2x^2 - 2 & if\ x \ge 1 \end{cases}$$

## Sample Problem 5: Graph each piecewise function.

a. $$f(x) = \begin{cases} -x^3 & \text{if } x < 0 \\ 3 & \text{if } 0 \le x < 1 \\ 2x^2 - 2 & \text{if } x \ge 1 \end{cases}$$



## Sample Problem 5: Graph each piecewise function.

**b.** $$f(x) = \begin{cases} 3x^2 & if \;\; x \leq -1 \\ -2 & if \; -1 < x < 2 \\ |x^2 - 1| & if \; x \geq 2 \end{cases}$$

## Sample Problem 5: Graph each piecewise function.

**b.** $$f(x) = \begin{cases} 3x^2 & if \;\; x \leq -1 \\ -2 & if \;\; -1 < x < 2 \\ |x^2 - 1| & if \; x \geq 2 \end{cases}$$