Parent Functions and Transformations

Unit 1 Lesson 5
Parent Functions and Transformations

Students will be able to:

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs.
Parent Functions and Transformations

**Key Vocabulary:**

- Parent function
- Transformation
- Translation
- Dilation
A family of functions is a group of functions with graphs that display one or more similar characteristics.

The Parent Function is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.
Family - Constant Function

Graph:

Rule $f(x) = c$
Domain $= (-\infty, \infty)$
Range $= [c]$
Family - Linear Function

Graph:

Rule \( f(x) = x \)
Domain = \( (-\infty, \infty) \)
Range = \( (-\infty, \infty) \)
Family - Quadratic Function

Graph:

Rule $f(x) = x^2$
Domain $= (-\infty, \infty)$
Range $= [0, \infty)$
Family - Cubic Function

Rule \( f(x) = x^3 \)
Domain = \((−\infty, \infty)\)
Range = \((−\infty, \infty)\)
Family - Square Root Function

Rule \( f(x) = \sqrt{x} \)
Domain = \([0, \infty)\)
Range = \([0, \infty)\)
**Family - Reciprocal Function**

**Rule**  \[ f(x) = \frac{1}{x} \]

**Domain**  \[ D = (-\infty, 0) \cup (0, \infty) \]

**Range**  \[ R = (-\infty, 0) \cup (0, \infty) \]
Family - Absolute Value Function

Graph:

Rule \( f(x) = |x| \)

\[ |x| = \begin{cases} 
-x & \text{if } x < 0 \\
 x & \text{if } x \geq 0 
\end{cases} \]

\( D = (-\infty, \infty) \)

\( R = [0, \infty) \)
Family - Greatest Integer Function

Rule $f(x) = \lfloor x \rfloor$
$D = (-\infty, \infty)$
$R = \text{All Integer}$
Transformations

A change in the size or position of a figure or graph of the function is called a transformation. **Rigid transformations** change only the position of the graph, leaving the size and shape unchanged. **Non rigid transformations** distort the shape of the graph.
# Parent Functions and Transformations

## Rigid transformations

### Vertical Translations

<table>
<thead>
<tr>
<th>Appearance in Function</th>
<th>Transformation of Graph</th>
<th>Transformation of Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) \rightarrow f(x) + a$</td>
<td>$a$ units up</td>
<td>$(x, y) \rightarrow (x, y + a)$</td>
</tr>
<tr>
<td>$f(x) \rightarrow f(x) - a$</td>
<td>$a$ units down</td>
<td>$(x, y) \rightarrow (x, y - a)$</td>
</tr>
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</table>
### Rigid Transformations

#### Horizontal Translations

<table>
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<th>Appearance in Function</th>
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</thead>
<tbody>
<tr>
<td>$f(x) \rightarrow f(x - b)$</td>
<td>$b$ units right</td>
<td>$(x, y) \rightarrow (x + b, y)$</td>
</tr>
<tr>
<td>$f(x) \rightarrow f(x + b)$</td>
<td>$b$ units left</td>
<td>$(x, y) \rightarrow (x - b, y)$</td>
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### Parent Functions and Transformations

**Rigid transformations**

#### Reflections in x-axes

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<tbody>
<tr>
<td>$f(x) \rightarrow -f(x)$</td>
<td>reflected in the $x$ axis</td>
<td>$(x, y) \rightarrow (x, -y)$</td>
</tr>
</tbody>
</table>
**Parent Functions and Transformations**

**Rigid transformations**

**Reflections in y-axes**

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<tr>
<td>$f(x) \rightarrow f(-x)$</td>
<td>reflected in the y axis</td>
<td>$(x, y) \rightarrow (-x, y)$</td>
</tr>
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## Non-rigid Transformations

### Vertical Dilations

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</tr>
</thead>
<tbody>
<tr>
<td>$f(x) \rightarrow cf(x)$</td>
<td>expanded vertically</td>
<td>$(x, y) \rightarrow (cx, y)$</td>
</tr>
<tr>
<td>$c &gt; 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) \rightarrow cf(x)$</td>
<td>compressed vertically</td>
<td>$(x, y) \rightarrow (cx, y)$</td>
</tr>
<tr>
<td>$0 &lt; c &lt; 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Parent Functions and Transformations

### Non rigid transformations

#### Horizontal Dilations

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<td>$f(x) \rightarrow f(dx)$</td>
<td>compressed horizontally</td>
<td>$(x, y) \rightarrow \left(\frac{x}{d}, y\right)$</td>
</tr>
<tr>
<td>$d &gt; 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) \rightarrow f(dx)$</td>
<td>expanded horizontally</td>
<td>$(x, y) \rightarrow \left(\frac{x}{d}, y\right)$</td>
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<td>$0 &lt; d &lt; 1$</td>
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Parent Functions and Transformations

Sample Problem 1: Identify the parent function and describe the transformations.

a. \( f(x) = (x - 1)^2 \)
Sample Problem 1: Identify the parent function and describe the transformations.

a. \( f(x) = (x - 1)^2 \)

\begin{align*}
\text{Parent: } & \quad f(x) = x^2 \\
\text{Transformation: } & \quad \text{Translation 1 unit right}
\end{align*}
Sample Problem 1: Identify the parent function and describe the transformations.

b. \( f(x) = x^3 - 5 \)
Sample Problem 1: Identify the parent function and describe the transformations.

b. \( f(x) = x^3 - 5 \)

**Parent:** \( f(x) = x^3 \)

**Transformation:** Translation 5 units down
Sample Problem 1: Identify the parent function and describe the transformations.

c. \( f(x) = -|x + 4| \)
Sample Problem 1: Identify the parent function and describe the transformations.

c. \( f(x) = -|x + 4| \)

**Parent:** \( f(x) = |x| \)

**Transformation:** Reflection in x-axis
Translation 4 units left
Sample Problem 1: Identify the parent function and describe the transformations.

d. \( f(x) = 3x^2 + 7 \)
Sample Problem 1: Identify the parent function and describe the transformations.

d. \( f(x) = 3x^2 + 7 \)

**Parent:** \( f(x) = x^2 \)

**Transformation:** Expand vertically by a factor of 3
Translation 7 units up
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

a. Quadratic - expanded horizontally by a factor of 2, translated 7 units up.
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

a. Quadratic - expanded horizontally by a factor of 2, translated 7 units up.

$$f(x) = \frac{1}{2} x^2 + 7$$
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

b. Cubic - reflected over the $x$ axis and translated 9 units down.
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

b. Cubic - reflected over the $x$ axis and translated 9 units down.

$$f(x) = -x^3 - 9$$
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

c. Absolute value - translated 3 units up, translated 8 units’ right.
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function \( f(x) \).

c. Absolute value - translated 3 units up, translated 8 units right.

\[
f(x) = |x - 8| + 3
\]
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

d. Reciprocal - translated 1 unit up.
Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

d. Reciprocal - translated 1 unit up.

$$f(x) = \frac{1}{x} + 1$$
Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

a. \( h(x) = 2(x - 3)^2 - 2 \)
Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

a. \( h(x) = 2(x - 3)^2 - 2 \)

\[
\begin{align*}
\text{Parent function} & \quad f(x) = x^2 \\
\text{Transformation:} & \\
\text{Compressed horizontally by a factor of 2} & \\
\text{Translated 2 units down} & \\
\text{Translated 3 units right} & \\
D & = (-\infty, \infty) \\
R & = (-2, \infty)
\end{align*}
\]
Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

b. \( h(x) = \sqrt{x - 5} + 3 \)
Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

b. \( h(x) = \sqrt{x - 5} + 3 \)

\( h(x) = \sqrt{x - 5} + 3 \)

Parent function \( f(x) = \sqrt{x} \)

Transformation:
Translated 3 units up
Translated 5 units right

\[ D = [5. \infty) \quad R = (3, \infty) \]
Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

c. \( h(x) = -|x + 4| - 1 \)
Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

c. \( h(x) = -|x + 4| - 1 \)

\[ h(x) = -|x + 4| - 1 \]

**Parent function** \( f(x) = |x| \)

**Transformation:**
- Reflected in the x axis
- Translated 1 unit down
- Translated 4 units left

\[ D = (-\infty, \infty) \quad R = (-\infty, -1] \]
Transformations with Absolute Value

\[ h(x) = |f(x)| \]

This transformation reflects any portion of the graph of \( f(x) \) that is below the \( x \)-axis so that it is above the \( x \)-axis.
Transformations with Absolute Value

\[ h(x) = f(|x|) \]

This transformation results, in the portion of the graph of \( f(x) \) that is to the left of the \( y \)-axis, being replaced by a reflection of the portion to the right of the \( y \)-axis.
Sample Problem 4: Graph each function.

a. \( f(x) = x^3 - 2x \)  
Graph \( h(x) = |x^3 - 2x| \)
Sample Problem 4: Graph each function.

a. \( f(x) = x^3 - 2x \)

Graph \( h(x) = |x^3 - 2x| \)
Sample Problem 4: Graph each function.

a. \( f(x) = x^3 - 2x \)  
\[ h(x) = |x^3 - 2x| \]
Sample Problem 4: Graph each function.

b. \( f(x) = \frac{1}{x - 3} \) \hspace{1cm} \text{Graph} \hspace{1cm} h(x) = \frac{1}{|x - 3|} \)
**Sample Problem 4:** Graph each function.

b. \( f(x) = \frac{1}{x - 3} \)  
   \[ f(x) = \frac{1}{x - 3} \]
Sample Problem 4: Graph each function.

b. \[ f(x) = \frac{1}{x - 3} \quad \text{Graph} \quad h(x) = \frac{1}{|x - 3|} \]
Graph a Piecewise-Defined Function
Sample Problem 5: Graph each piecewise function.

a. $f(x) = \begin{cases} 
-x^3 & \text{if } x < 0 \\
3 & \text{if } 0 \leq x < 1 \\
2x^2 - 2 & \text{if } x \geq 1 
\end{cases}$
Sample Problem 5: Graph each piecewise function.

a. \[ f(x) = \begin{cases} 
-x^3 & \text{if } x < 0 \\
3 & \text{if } 0 \leq x < 1 \\
2x^2 - 2 & \text{if } x \geq 1 
\end{cases} \]
Parent Functions and Transformations

Sample Problem 5: Graph each piecewise function.

b. \( f(x) = \begin{cases} 
3x^2 & \text{if } x \leq -1 \\
-2 & \text{if } -1 < x < 2 \\
|x^2 - 1| & \text{if } x \geq 2 
\end{cases} \)
Sample Problem 5: Graph each piecewise function.

b. \( f(x) = \begin{cases} 
3x^2 & \text{if } x \leq -1 \\
-2 & \text{if } -1 < x < 2 \\
|x^2 - 1| & \text{if } x \geq 2 
\end{cases} \)