**A family of functions** is a group of functions with graphs that display one or more similar characteristics.

**The Parent Function** is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

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| **Family - Constant Function** | **Family - Linear Function** | **Family - Quadratic Function** |
| **Graph** | **Graph** | **Graph** |
| **Rule** $f\left(x\right)=c $**Domain**$ = (-\infty ,\infty )$**Range** $=\left[c\right]$ | **Rule** $f\left(x\right)=x$**Domain**$=\left(-\infty ,\infty \right)$**Range**$ =\left(-\infty ,\infty \right)$$ $ | **Rule** $f\left(x\right)=x^{2}$**Domain**$= (-\infty ,\infty )$**Range** $ =[0,\infty )$ |
| **Family - Cubic Function** | **Family - Square Root Function** | **Family - Reciprocal Function** |
| **Graph** | **Graph** | **Graph** |
| **Rule** $f\left(x\right)=x^{3} $**Domain**$=(-\infty ,\infty )$**Range** $=\left(-\infty ,\infty \right)$ | **Rule** $f\left(x\right)=\sqrt{x}$**Domain**$= [0,\infty )$**Range**$ =[0,\infty )$$ $ | **Rule** $ f\left(x\right)=\frac{1}{x}$**Domain**$=(-\infty ,0)∪(0,\infty )$**Range** $ =(-\infty ,0)∪(0,\infty )$ |

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| **Family – Absolut Value Function** | **Family -**  **Greatest Integer Function**  |
| **Graph** | **Graph** |
| **Rule** $f\left(x\right)=\left|x\right|=\left\{\begin{array}{c}-x if x<0\\ x if x\geq 0\end{array}\right. $**Domain**$=(-\infty ,\infty )$**Range** $=[0,\infty )$ | **Rule** $f\left(x\right)=\left⟦x\right⟧$**Domain**$=(-\infty ,\infty )$**Range**$ All Integer$$ $ |

***Transformations***

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| ***Transformations*** A change in the size or position of a figure or graph of the function is called a transformation. |
| ***Rigid transformations*** change only the position of the graph, leaving the size and shape unchanged. |
|  | **Appearance in Function** | **Transformation of Graph** | **Transformation of Point** |
| ***Vertical Translations*** | $$f\left(x\right)\rightarrow f\left(x\right)+a$$$$f\left(x\right)\rightarrow f\left(x\right)-a$$ | $$a units up$$$$a units down$$ | $$\left(x,y\right)\rightarrow \left(x,y+a\right)$$$$(x,y)\rightarrow (x,y-a)$$ |
| ***Horizontal Translations*** | $$f\left(x\right)\rightarrow f\left(x-b\right)$$$$f\left(x\right)\rightarrow f\left(x+b\right)$$ | $$b units right$$$$b units left$$ | $$\left(x,y\right)\rightarrow \left(x+b,y\right)$$$$(x,y)\rightarrow (x-b,y)$$ |
| ***Reflections in x-axes*** | $$f\left(x\right)\rightarrow -f\left(x\right)$$ | $$reflected in the x axis$$ | $$\left(x,y\right)\rightarrow \left(x,-y\right)$$ |
| ***Reflections in y-axes*** | $$f\left(x\right)\rightarrow f\left(-x\right)$$ | $$reflected in the y axis$$ | $$\left(x,y\right)\rightarrow \left(-x,y\right)$$ |
| ***Non rigid transformations*** distort the shape of the graph. |
|  | **Appearance in Function** | **Transformation of Graph** | **Transformation of Point** |
| ***Vertical***  ***Dilations*** | $$f\left(x\right)\rightarrow cf\left(x\right) c>1$$$$f\left(x\right)\rightarrow cf\left(x\right) 0<c<1$$ | $$expanded vertically$$$$compressed vertically $$ | $$\left(x,y\right)\rightarrow \left(cx,y\right)$$ |
| ***Horizontal***  ***Dilations*** | $$f\left(x\right)\rightarrow f\left(dx\right) d>1$$$$f\left(x\right)\rightarrow f\left(dx\right) 0<d<1$$ | $$compressed horizontally$$$$expanded horizontally$$ | $$\left(x,y\right)\rightarrow \left(\frac{x}{d},y\right)$$ |

**Sample Problem 1: Identify the parent function and describe the transformations.**

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| **a.**  | $$f\left(x\right)=(x-1)^{2} $$ | **Parent :**$ f\left(x\right)=x^{2}$**Transformation:**  Translation 1 unit right |
| **b.**  | $$f\left(x\right)=x^{3}-5$$ | **Parent :**$ f\left(x\right)=x^{3}$**Transformation:**  Translation 5 units down |
| **c.**  | $$f\left(x\right)=-\left|x+4\right|$$ | **Parent :**$ f\left(x\right)=\left|x\right|$**Transformation:**  Reflection in x-axis Translation 4 units left |
| **d.** | $$f\left(x\right)=3x^{2}+7$$ | **Parent :**$ f\left(x\right)=x^{2}$**Transformation:**  Expand vertically by a factor of 3 Translation 7 units up |

**Sample Problem 2**: **Given the parent function and a description of the transformation, write the equation of the transformed function**$ f\left(x\right)$ **.**

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| **a.**  | **Quadratic** - **expanded horizontally by a factor of 2, translated 7 units up.**  | $$f\left(x\right)=\frac{1}{2}x^{2}+7$$ |
| **b.**  | **Cubic - reflected over the x axis and translated 9 units down.**  | $$f\left(x\right)=-x^{3}-9$$ |
| **c.** | **Absolute value - translated 3 units up, translated 8 units right.** | $$f\left(x\right)=\left|x-8\right|+3$$ |
| **d.**  | **Reciprocal -** **translated 1 unit up.** | $$f\left(x\right)=\frac{1}{x}+1$$ |

**Sample Problem 3:**  **Use the graph of parent function to graph each function. Find the domain and the range of the new function.**

|  |  |  |  |
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| **a.**  | $$h\left(x\right)=2\left(x-3\right)^{2}-2$$ |  |  |
|  | $h\left(x\right)=2\left(x-3\right)^{2}-2$**Parent function** $f\left(x\right)=x^{2}$**Transformation:**Expand vertically by a factor of 2Translated 2 units downTranslated 3 units right$$D=\left(-\infty ,\infty \right)$$$$R=\left(-2,\infty \right)$$ |  |  |
| **b.** | $$h\left(x\right)=\sqrt{x-5}+3$$ |  |  |
|  | $h\left(x\right)=\sqrt{x-5}+3$**Parent function** $f\left(x\right)=\sqrt{x}$**Transformation:** Translated 3 units upTranslated 5 units right$$D=[5.\infty )$$$$R=\left(3,\infty \right)$$ |  |  |
| **c.** | $$h\left(x\right)=-\left|x+4\right|-1$$ |  |  |
|  | $h\left(x\right)=-\left|x+4\right|-1$$ $**Parent function** $f\left(x\right)=\left|x\right|$**Transformation:** Reflected in the x axis Translated 1 unit downTranslated 4 units left$$D=(-\infty .\infty )$$$$R=(-\infty ,-1]$$ |  |  |

**Transformations with Absolute Value**

$$h\left(x\right)=\left|f(x)\right|$$

 This transformation reflects any portion of the graph of $f(x)$ that is below the $x$ -axis so that it is above the $x$ -axis.

$$h\left(x\right)=f(\left|x\right|)$$

This transformation results, in the portion of the graph of $f\left(x\right) $that is to the left of the $y$-axis, being replaced by a reflection of the portion to the right of the$ y$ -axis.

**Sample Problem 4:**  **Graph each function.**

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| **a.**  | $$f\left(x\right)=x^{3}-2x Graph h\left(x\right)=\left|x^{3}-2x\right|$$ |  |  |
|  | $f\left(x\right)=x^{3}-2x$$h\left(x\right)=\left|x^{3}-2x\right|$ |  |  |
| **b.** | $$f\left(x\right)=\frac{1}{x-3} Graph h\left(x\right)=\frac{1}{\left|x-3\right|}$$ |  |  |
|  | $f\left(x\right)=\frac{1}{x-3}$$h\left(x\right)=\frac{1}{\left|x-3\right|}$ |  |  |

**Graph a Piecewise-Defined Function**

**Sample Problem 5:**  **Graph each piecewise function.**

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| **a.** | $$f\left(x\right)=\left\{\begin{array}{c}-x^{3} if x<0\\3 if 0\leq x<1\\2x^{2}-2 if x\geq 1\end{array}\right.$$ |  |
| **b.** | $$f\left(x\right)=\left\{\begin{array}{c}3x^{2} if x\leq -1\\-2 if -1<x<2\\\left|x^{2}-1\right| if x\geq 2\end{array}\right.$$ |  |