A family of functions is a group of functions with graphs that display one or more similar characteristics.

The Parent Function is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

**Family - Constant Function**

- **Graph**
  - Rule \( f(x) = c \)
  - Domain = \((-\infty, \infty)\)
  - Range = \([c]\)

**Family - Linear Function**

- **Graph**
  - Rule \( f(x) = x \)
  - Domain = \((-\infty, \infty)\)
  - Range = \((-\infty, \infty)\)

**Family - Quadratic Function**

- **Graph**
  - Rule \( f(x) = x^2 \)
  - Domain = \((-\infty, \infty)\)
  - Range = \([0, \infty)\)

**Family - Cubic Function**

- **Graph**
  - Rule \( f(x) = x^3 \)
  - Domain = \((-\infty, \infty)\)
  - Range = \((-\infty, \infty)\)

**Family - Square Root Function**

- **Graph**
  - Rule \( f(x) = \sqrt{x} \)
  - Domain = \([0, \infty)\)
  - Range = \([0, \infty)\)

**Family - Reciprocal Function**

- **Graph**
  - Rule \( f(x) = \frac{1}{x} \)
  - Domain = \((-\infty, 0) \cup (0, \infty)\)
  - Range = \((-\infty, 0) \cup (0, \infty)\)
Parent Functions and Transformations  Guided Notes

**Family – Absolut Value Function**

Graph

Rule \( f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \)

Domain = \((-\infty, \infty)\)
Range = \([0, \infty)\)

**Family - Greatest Integer Function**

Graph

Rule \( f(x) = \lfloor x \rfloor \)  
Domain = \((-\infty, \infty)\)  
Range = *All Integer*

**Transformations**

A change in the size or position of a figure or graph of the function is called a transformation.

**Rigid transformations** change only the position of the graph, leaving the size and shape unchanged.

<table>
<thead>
<tr>
<th>Transformation of Graph</th>
<th>Transformation of Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \text{ units up} )</td>
<td>((x, y) \rightarrow (x, y + a))</td>
</tr>
<tr>
<td>( a \text{ units down} )</td>
<td>((x, y) \rightarrow (x, y - a))</td>
</tr>
<tr>
<td>( b \text{ units right} )</td>
<td>((x, y) \rightarrow (x + b, y))</td>
</tr>
<tr>
<td>( b \text{ units left} )</td>
<td>((x, y) \rightarrow (x - b, y))</td>
</tr>
</tbody>
</table>

**Non rigid transformations** distort the shape of the graph.

<table>
<thead>
<tr>
<th>Transformation of Graph</th>
<th>Transformation of Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>expanded vertically</td>
<td>((x, y) \rightarrow (cx, y))</td>
</tr>
<tr>
<td>compressed vertically</td>
<td>((x, y) \rightarrow \left( \frac{x}{d}, y \right))</td>
</tr>
</tbody>
</table>

Horizontal Dilation

Vertical Dilation

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Parent Functions and Transformations  Guided Notes

Sample Problem 1: Identify the parent function and describe the transformations.

a. \( f(x) = (x - 1)^2 \)
   Parent: Quadratic
   Transformation: expanded horizontally by a factor of 2, translated 7 units up.

b. \( f(x) = x^3 - 5 \)
   Parent: Cubic
   Transformation: reflected over the x axis and translated 9 units down.

c. \( f(x) = -|x + 4| \)
   Parent: Absolute value
   Transformation: translated 3 units up, translated 8 units' right.

d. \( f(x) = 3x^2 + 7 \)
   Parent: Quadratic
   Transformation: translated 1 unit up.

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function \( f(x) \).

a. Quadratic - expanded horizontally by a factor of 2, translated 7 units up.

b. Cubic - reflected over the x axis and translated 9 units down.

c. Absolute value - translated 3 units up, translated 8 units’ right.

d. Reciprocal - translated 1 unit up.

Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

a. \( h(x) = 2(x - 3)^2 - 2 \)
Parent Functions and Transformations Guided Notes

b. \( h(x) = \sqrt{x - 5} + 3 \)

c. \( h(x) = -|x + 4| - 1 \)

Transformations with Absolute Value

\( h(x) = |f(x)| \)

This transformation reflects any portion of the graph of \( f(x) \) that is below the \( x \)-axis so that it is above the \( x \)-axis.

\( h(x) = f(|x|) \)

This transformation results, in the portion of the graph of \( f(x) \) that is to the left of the \( y \)-axis, being replaced by a reflection of the portion to the right of the \( y \)-axis.
Sample Problem 4: Graph each function.

a. \( f(x) = x^3 - 2x \)  \hspace{1cm} \text{Graph} \hspace{1cm} h(x) = |x^3 - 2x|

\[ \begin{align*} 
\text{Graph of } f(x) & \hspace{1cm} \text{Graph of } h(x) 
\end{align*} \]

b. \( f(x) = \frac{1}{x - 3} \)  \hspace{1cm} \text{Graph} \hspace{1cm} h(x) = \frac{1}{|x - 3|}

\[ \begin{align*} 
\text{Graph of } f(x) & \hspace{1cm} \text{Graph of } h(x) 
\end{align*} \]
Sample Problem 5: Graph each piecewise function.

a. \[ f(x) = \begin{cases} \ -x^3 & \text{if } x < 0 \\ 3 & \text{if } 0 \leq x < 1 \\ 2x^2 - 2 & \text{if } x \geq 1 \end{cases} \]

b. \[ f(x) = \begin{cases} \ 3x^2 & \text{if } x \leq -1 \\ -2 & \text{if } -1 < x < 2 \\ |x^2 - 1| & \text{if } x \geq 2 \end{cases} \]