

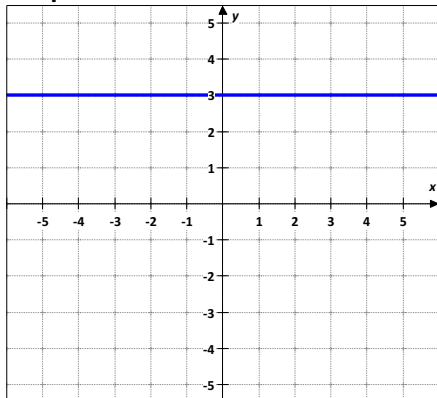
# Parent Functions and Transformations Guided Notes

A **family of functions** is a group of functions with graphs that display one or more similar characteristics.

The **Parent Function** is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

## Family - Constant Function

### Graph



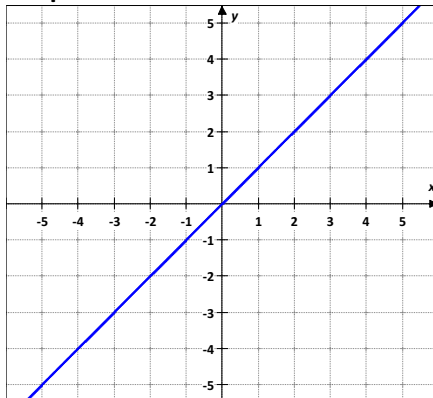
Rule  $f(x) = c$

Domain =  $(-\infty, \infty)$

Range =  $[c]$

## Family - Linear Function

### Graph



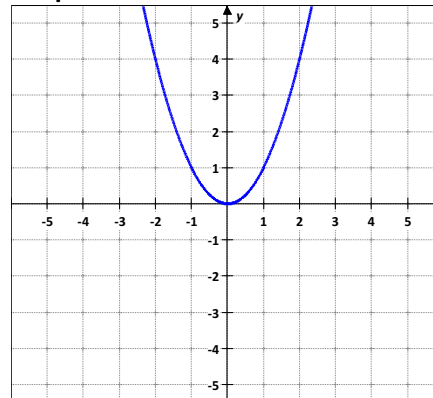
Rule  $f(x) = x$

Domain =  $(-\infty, \infty)$

Range =  $(-\infty, \infty)$

## Family - Quadratic Function

### Graph



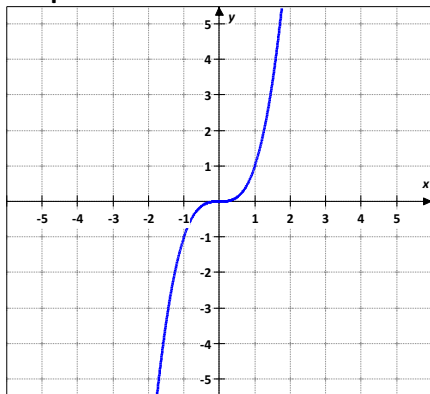
Rule  $f(x) = x^2$

Domain =  $(-\infty, \infty)$

Range =  $[0, \infty)$

## Family - Cubic Function

### Graph



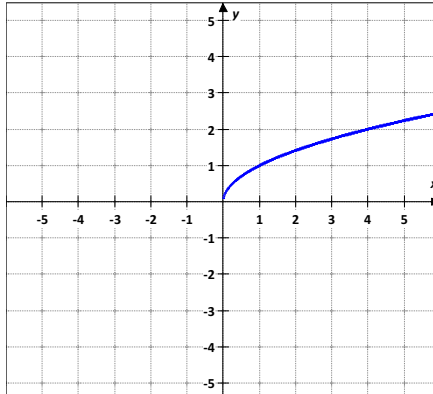
Rule  $f(x) = x^3$

Domain =  $(-\infty, \infty)$

Range =  $(-\infty, \infty)$

## Family - Square Root Function

### Graph



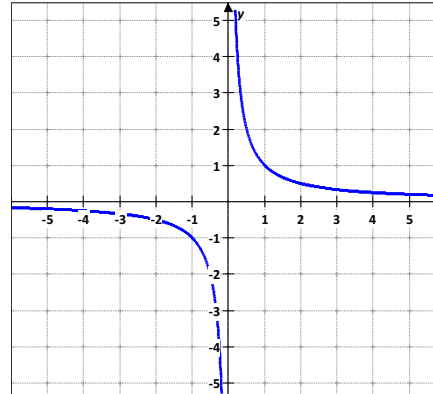
Rule  $f(x) = \sqrt{x}$

Domain =  $[0, \infty)$

Range =  $[0, \infty)$

## Family - Reciprocal Function

### Graph



Rule  $f(x) = \frac{1}{x}$

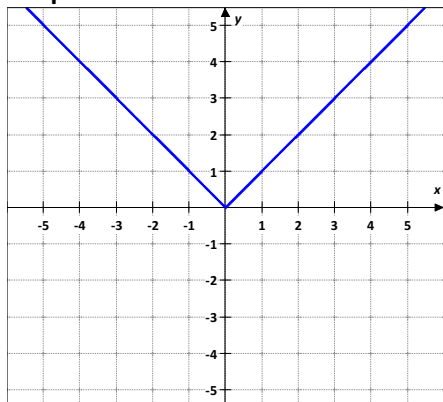
Domain =  $(-\infty, 0) \cup (0, \infty)$

Range =  $(-\infty, 0) \cup (0, \infty)$

# Parent Functions and Transformations Guided Notes

## Family – Absolut Value Function

### Graph



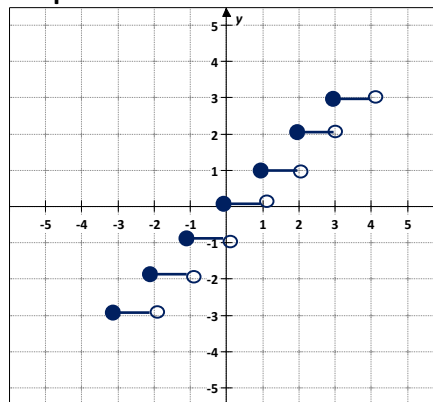
Rule  $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

Domain =  $(-\infty, \infty)$

Range =  $[0, \infty)$

## Family - Greatest Integer Function

### Graph



Rule  $f(x) = \lfloor x \rfloor$

Domain =  $(-\infty, \infty)$

Range *All Integer*

## Transformations

### Transformations

A change in the size or position of a figure or graph of the function is called a transformation.

**Rigid transformations** change only the position of the graph, leaving the size and shape unchanged.

|                                | Appearance in Function                                     | Transformation of Graph                     | Transformation of Point  |
|--------------------------------|--|---|--|
| <b>Vertical Translations</b>   | $f(x) \rightarrow f(x) + a$<br>$f(x) \rightarrow f(x) - a$ | <i>a</i> units up<br><i>a</i> units down    | $(x, y) \rightarrow (x, y + a)$<br>$(x, y) \rightarrow (x, y - a)$ |
| <b>Horizontal Translations</b> | $f(x) \rightarrow f(x - b)$<br>$f(x) \rightarrow f(x + b)$ | <i>b</i> units right<br><i>b</i> units left | $(x, y) \rightarrow (x + b, y)$<br>$(x, y) \rightarrow (x - b, y)$ |
| <b>Reflections in x-axes</b>   | $f(x) \rightarrow -f(x)$                                   | reflected in the <i>x</i> axis              | $(x, y) \rightarrow (x, -y)$                                       |
| <b>Reflections in y-axes</b>   | $f(x) \rightarrow f(-x)$                                   | reflected in the <i>y</i> axis              | $(x, y) \rightarrow (-x, y)$                                       |

**Non rigid transformations** distort the shape of the graph.

|                             | Appearance in Function   | Transformation of Graph                          | Transformation of Point                          |
|-----------------------------|--|--|--|
| <b>Vertical Dilations</b>   | $f(x) \rightarrow cf(x)$ $c > 1$<br>$f(x) \rightarrow cf(x)$ $0 < c < 1$ | expanded vertically<br>compressed vertically     | $(x, y) \rightarrow (cx, y)$                     |
| <b>Horizontal Dilations</b> | $f(x) \rightarrow f(dx)$ $d > 1$<br>$f(x) \rightarrow f(dx)$ $0 < d < 1$ | compressed horizontally<br>expanded horizontally | $(x, y) \rightarrow \left(\frac{x}{d}, y\right)$ |

# Parent Functions and Transformations Guided Notes

**Sample Problem 1:** Identify the parent function and describe the transformations.

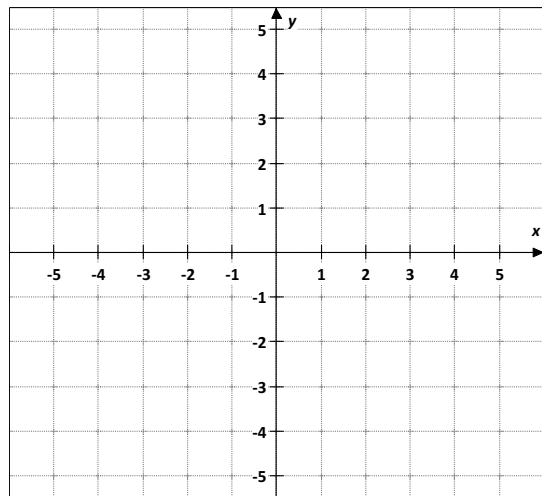
- a.  $f(x) = (x - 1)^2$   
Parent : \_\_\_\_\_  
Transformation: \_\_\_\_\_
- b.  $f(x) = x^3 - 5$   
Parent : \_\_\_\_\_  
Transformation: \_\_\_\_\_
- c.  $f(x) = -|x + 4|$   
Parent \_\_\_\_\_  
Transformation: \_\_\_\_\_
- d.  $f(x) = 3x^2 + 7$   
Parent : \_\_\_\_\_  
Transformation: \_\_\_\_\_

**Sample Problem 2:** Given the parent function and a description of the transformation, write the equation of the transformed function  $f(x)$ .

- a. Quadratic - expanded horizontally by a factor of 2, translated 7 units up.
- b. Cubic - reflected over the x axis and translated 9 units down.
- c. Absolute value - translated 3 units up, translated 8 units' right.
- d. Reciprocal - translated 1 unit up.

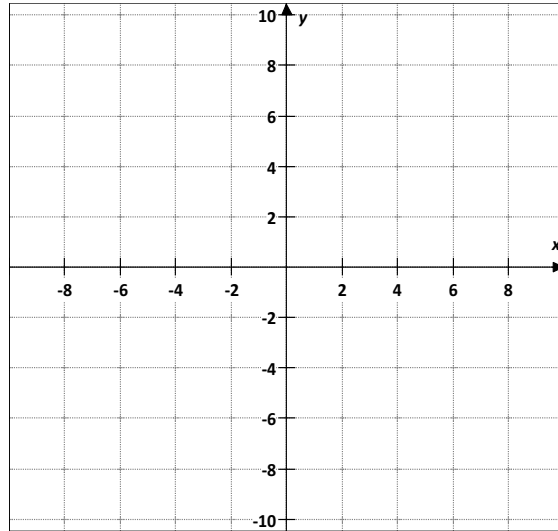
**Sample Problem 3:** Use the graph of parent function to graph each function. Find the domain and the range of the new function.

a.  $h(x) = 2(x - 3)^2 - 2$

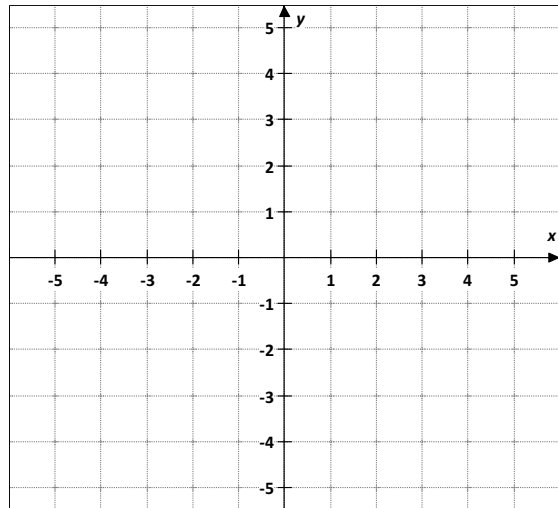


# Parent Functions and Transformations Guided Notes

b.  $h(x) = \sqrt{x - 5} + 3$



c.  $h(x) = -|x + 4| - 1$



## Transformations with Absolute Value

$$h(x) = |f(x)|$$

This transformation reflects any portion of the graph of  $f(x)$  that is below the  $x$ -axis so that it is above the  $x$ -axis.

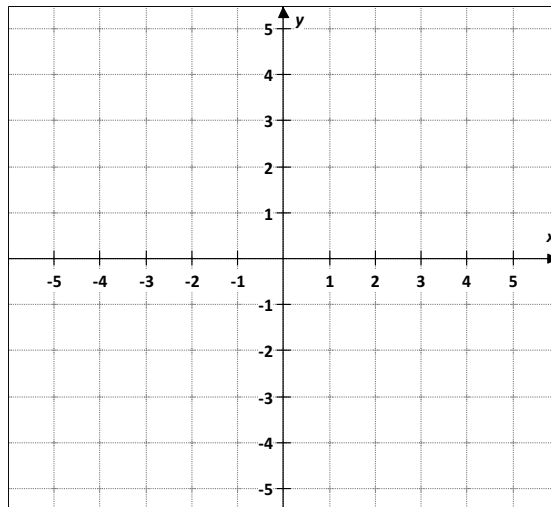
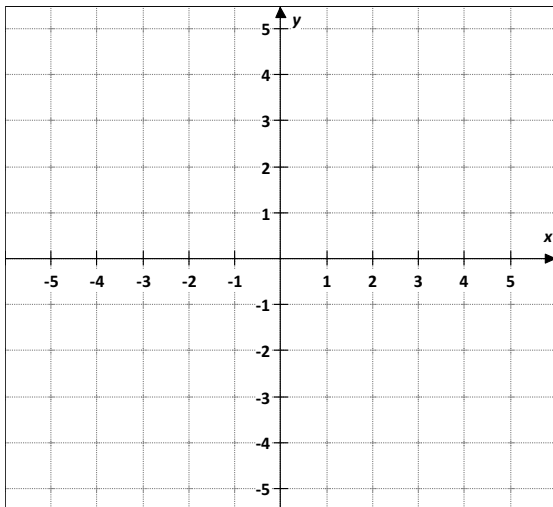
$$h(x) = f(|x|)$$

This transformation results, in the portion of the graph of  $f(x)$  that is to the left of the  $y$ -axis, being replaced by a reflection of the portion to the right of the  $y$ -axis.

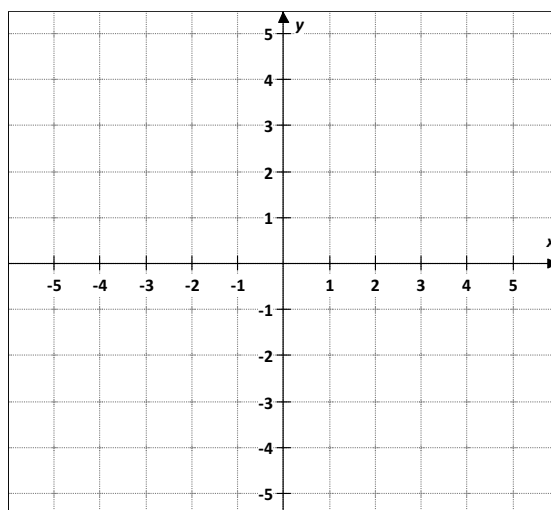
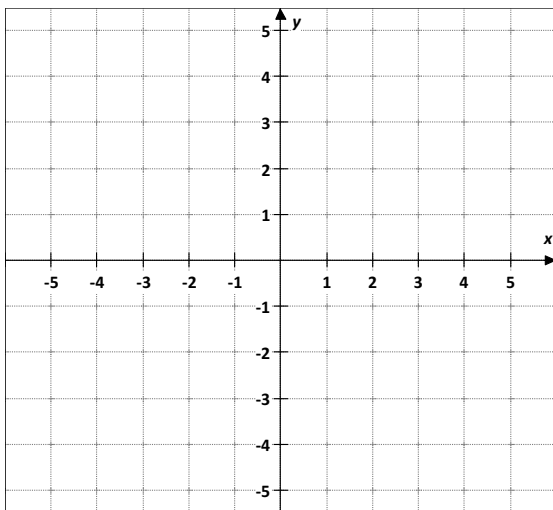
# Parent Functions and Transformations Guided Notes

**Sample Problem 4:** Graph each function.

a.  $f(x) = x^3 - 2x$  Graph  $h(x) = |x^3 - 2x|$



b.  $f(x) = \frac{1}{x-3}$  Graph  $h(x) = \frac{1}{|x-3|}$

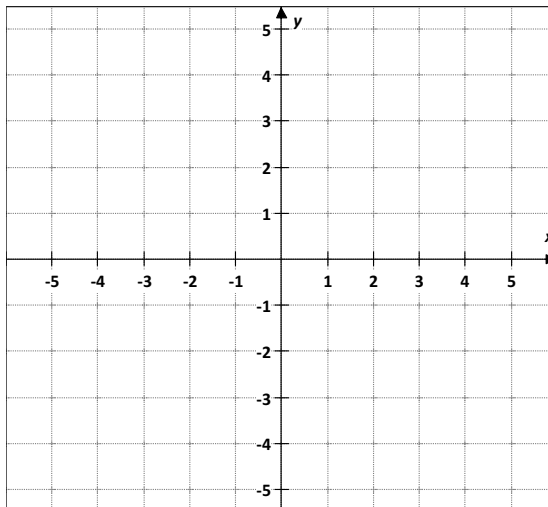


# Parent Functions and Transformations Guided Notes

## Graph a Piecewise-Defined Function

**Sample Problem 5:** Graph each piecewise function.

a. 
$$f(x) = \begin{cases} -x^3 & \text{if } x < 0 \\ 3 & \text{if } 0 \leq x < 1 \\ 2x^2 - 2 & \text{if } x \geq 1 \end{cases}$$



b. 
$$f(x) = \begin{cases} 3x^2 & \text{if } x \leq -1 \\ -2 & \text{if } -1 < x < 2 \\ |x^2 - 1| & \text{if } x \geq 2 \end{cases}$$

