Name: ____ Period: __________ Date: _______________

Extrema and Average Rates of Change Guided Notes

Increasing and Decreasing Behavior

A function f is increasing on an interval I if and only if for every a and b contained in I, $(a) < f(b)$, whenever $a < b$.

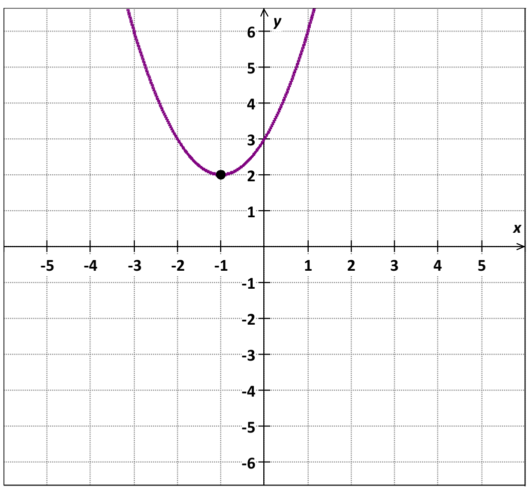
A function f is decreasing on an interval I if and only if for every a and b contained in I, $f(a) > f(b)$ whenever $a < b$.

A function f remains constant on an interval I if and only if for every a and b contained in I, $f(a) = f(b)$ whenever $a < b$.

Points in the domain of a function where the function changes from increasing to decreasing or from decreasing to increasing are called **critical points.** At these points, a line drawn tangent to the curve is horizontal or vertical.

Sample Problem 1: Use the graph of each function to estimate intervals on which the function is increasing, decreasing, or constant. Support the answer numerically.

a. $f(x) = x^2 + 2x + 3$

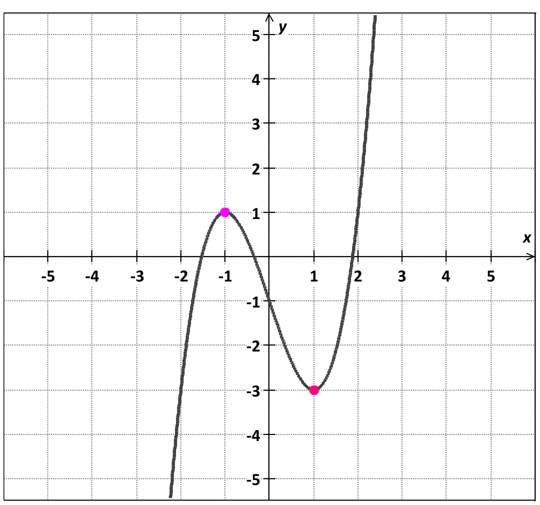


From the graph, it appears that:
A function $x^2 + 2x + 3$ is decreasing for $x < -1$.
A function $x^2 + 2x + 3$ is increasing for $x > -1$.
The critical point is $(-1, 2)$.

x	-4	-2	-1	0	1	2
y	11	3	2	3	6	11

The table supports this conjecture.

b. $f(x) = x^3 - 3x - 1$



From the graph, it appears that:

A function $x^3 - 3x - 1$ is increasing for $x < -1$ *and* $x > 1$.
A function $x^3 - 3x - 1$ is decreasing for $-1 < x < 1$.
The critical points are $(-1, 1)$ *and* $(1, 3)$.

$x < -1$

x	-5	-4	-3	-2	-1.5
y	-111	-53	-19	-3	-0.125

$-1 < x < 1$

x	-0.9	-0.5	0	0.5	0.9
y	0.971	-0.375	-1	-2.375	-2.971

$x > 1$

x	1.1	1.5	2	3	4
y	-2.969	-2.125	-1	17	51

The tables support this conjecture.

Name: ____ Period: ___________ Date: ________________

Extrema and Average Rates of Change Guided Notes

Extrema are critical points at which a function changes its increasing or decreasing behavior. At these points, the function has a maximum or a minimum value, either relative or absolute.

The greatest value that a function assumes over its domain is called the **absolute maximum**.

The least value that a function assumes over its domain is called the **absolute minimum**.

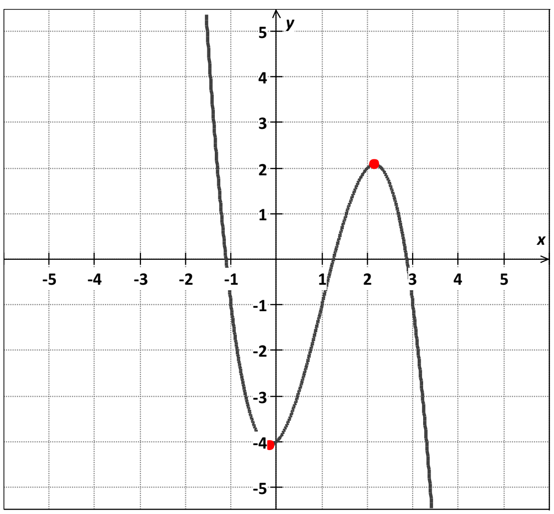
A relative maximum value of a function may not be the greatest value of f on the domain, but it is the greatest value on some interval of the domain.

A relative minimum value of a function is the least value on some interval of the domain.

A point of inflection can also be a critical point. At these points, the graph changes its shape, but not it's increasing or decreasing behavior. Instead, the curve changes from being bent upward to being bent downward, or vice versa.

Sample Problem 2: **Estimate and classify the extrema for the graph of each function. Support the answers numerically.**

a. $f(x) = -x^3 + 3x^2 + x - 4$



From the graph, it appears that:

$f(x)$ has relative minimum in $x = -0.15$
$f(x)$ has relative maximum in $x = 2.15$

$\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$
$f(x)$ has no absolute maxima and absolute minima.

$x < -0.15$ or $(-\infty, -0.15)$

x	-2	-1.44	-1	-0.5	-0.25
y	14	3.766	-1.1	-3.625	-4.1

$-0.15 < x < 2.15$ or $(-0.15; 2.15)$

x	-0.1	0	0.5	1.44	1.92
y	-4.069	-4	-2.94	0.675	1.9

$x > 2.15$ or $(2.15; \infty)$

x	2.5	3	3.5	4	4.5
y	1.625	-1	-6.625	-16	-29.88

The tables support this conjecture.

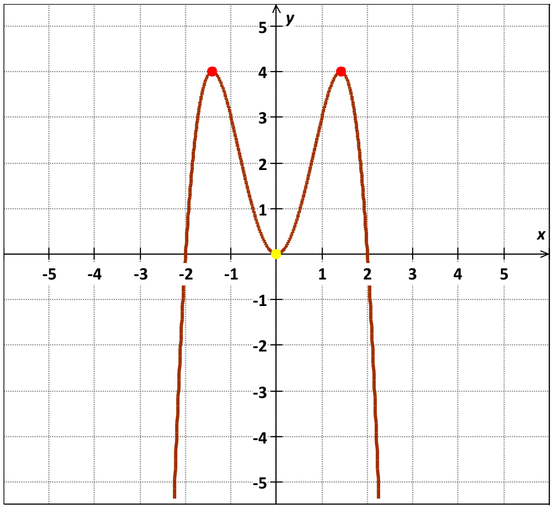
For interval $(-\infty, -0.15)$ the function is decreasing.
In $x = -0.15$, $f(x)$ has relative minimum.
For interval $(-0.15; 2.15)$ the function is increasing.
In $x = 2.15$, $f(x)$ has relative maximum.
For interval $(2.15; \infty)$ the function is decreasing.

PreCalculusCoach.com

Name: ____ Period: __________ Date: _______________

Extrema and Average Rates of Change Guided Notes

b. $f(x) = -x^4 + 4x^2$



For interval $(-\infty; -1.41)$ the function is increasing.
In $x = -1.41$, $f(x)$ has absolute maximum.
For interval $(-1.41; 0)$ the function is decreasing.
In $x = 0$, $f(x)$ has relative minimum.
For interval $(0; 1.41)$ the function is increasing.
In $x = 1.41$, $f(x)$ has absolute maximum.
For interval $(1.41; \infty)$ the function is decreasing.

From the graph, it appears that:

$f(x)$ has relative minimum in $x = 0$
$f(x)$ has absolute maximum in $x = -1.41$ and $x = 1.41$

$\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$
$f(x)$ has no absolute minima.

$x < -1.41$ *or* $(-\infty; -1.41)$

x	-3	-2.5	-2	-1.5
y	-45	-14.06	0	3.93

$-1.41 < x < 0$ *or* $(-1.41; 0)$

x	-1.30	-1.20	-1.10	-1
y	3.90	3.67	3.35	3

$0 < x < 1.41$ *or* $(0; 1.41)$

x	0.50	0.70	1.0	1.20
y	0.94	1.72	3	3.67

$x > 1.41$ *or* $(1.41; \infty)$

x	1.5	2	2.5	3.0
y	3.93	0	-14.06	-45

The tables support this conjecture.

Average Rate of Change

The average rate of change between any two points on the graph of f is the slope of the line through those points.

The line through two points on a curve is called a secant line. The slope of the secant line is denoted m_{sec}.

The average rate of change on the interval $[x_1; x_2]$ is: $m_{sec} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$

When the average rate of change over an interval is positive, the function increases on average over that interval.

When the average rate of change is negative, the function decreases on average over that interval.

Name: ____ Period: __________ Date: _______________

Extrema and Average Rates of Change Guided Notes

Sample Problem 3: **Find the average rate of change of each function on the given interval.**

a. $f(x) = 2x^2 + 2x - 1 \quad [1; 3]$

$f(x) = 2x^2 + 2x - 1 \quad [1; 3]$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(3) - f(1)}{3 - 1} =$$

$$= \frac{2 * 3^2 + 2 * 3 - 1 \; - (2 * 1^2 + 2 * 1 - 1)}{2} =$$

$$= \frac{2 * 9 + 2 * 3 - 1 \; - (2 * 1 + 2 * 1 - 1)}{2} =$$

$$= \frac{18 + 6 - 1 \; - (2 + 2 - 1)}{2}$$

$$= \frac{24 \; - 4}{2} =$$

$$= \frac{20}{2} = \mathbf{10}$$

The average rate of change on the interval $[1; 3]$ is 10 .

b. $f(x) = \dfrac{x - 5}{2 - x} \quad [0; -1]$

$f(x) = \dfrac{x - 5}{2 - x} \quad [0; -1]$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(-1) - f(0)}{-1 - 0} =$$

$$= \frac{\frac{-1 - 5}{2 - (-1)} - \frac{0 - 5}{2 - 0}}{-1} =$$

$$= \frac{\frac{-6}{3} - \frac{-5}{2}}{-1} =$$

$$= \frac{6}{3} - \frac{5}{2} =$$

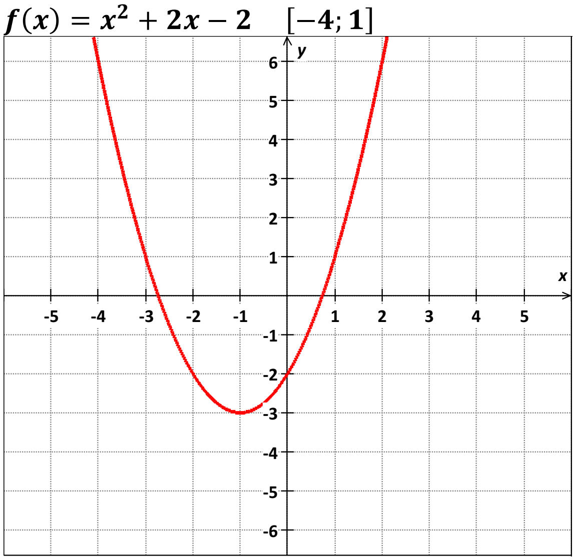
$$= -\frac{1}{2}$$

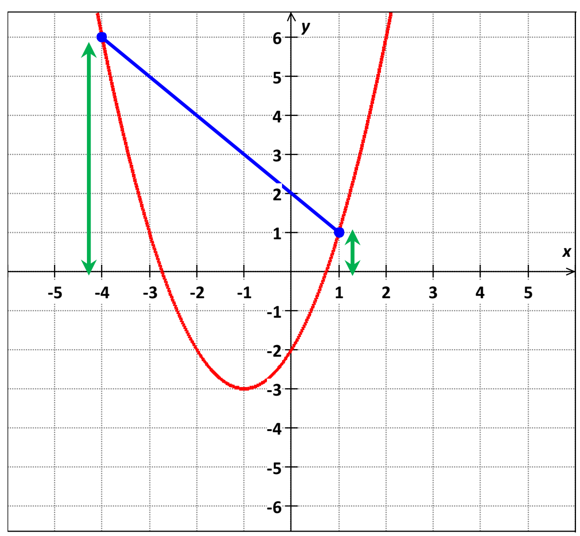
The average rate of change on the interval $[0; -1]$ is $-\frac{1}{2}$.

Computing Average Rate of Change from a Graph

Sample Problem 4: **Find the average rate of change of a function on the given interval.**

a. $f(x) = x^2 + 2x - 2 \quad [-4; 1]$





$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(-4)}{1 - (-4)} = \frac{1 - 6}{1 - (-4)} = \frac{-5}{5} = \mathbf{-1}$$

Name: ____ Period: __________ Date: _______________

ExtremA and Average Rates of Change Guided Notes