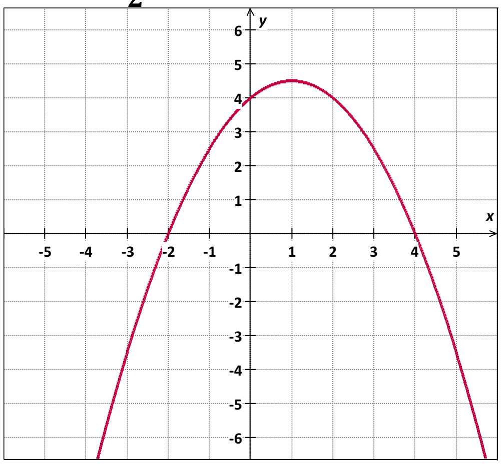
Name: ____ Period: __________ Date: _______________

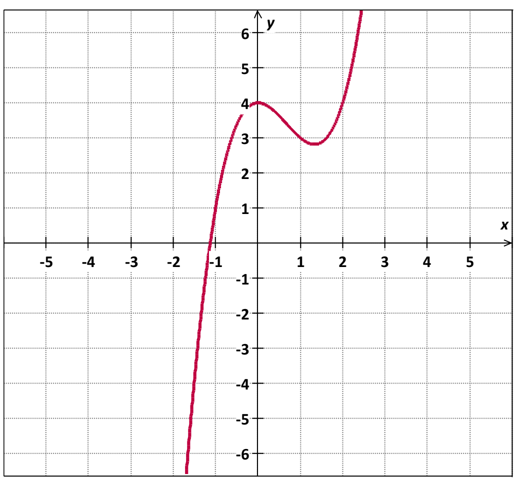
Extrema and Average Rates of Change Assignment

Use the graph of each function to estimate intervals on which the function is increasing, decreasing, or constant. Support the answer numerically.

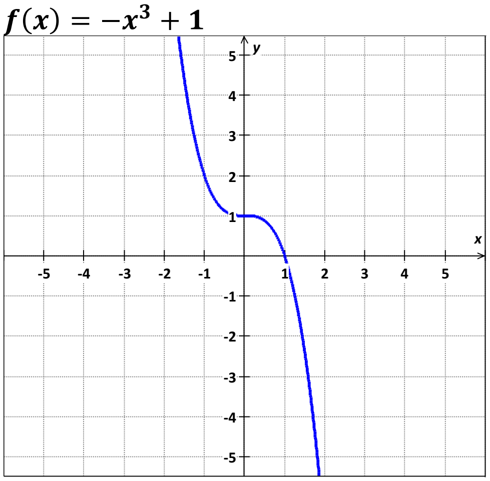
1. $f(x) = -\frac{1}{2}x^2 + x + 4$



2. $f(x) = x^3 - 2x^2 + 4$



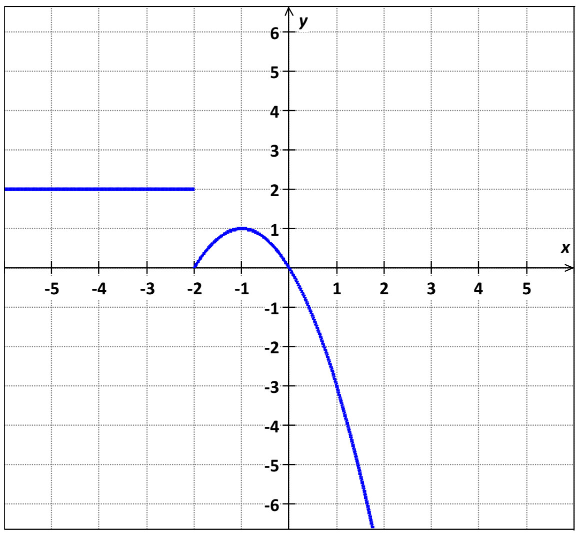
3. $f(x) = -x^3 + 1$



Name: ____ Period: ___________ Date: ________________

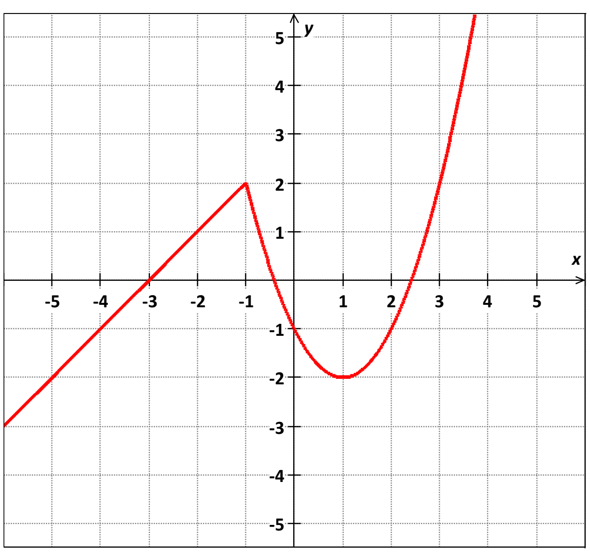
Extrema and Average Rates of Change Assignment

4. $f(x) = \begin{cases} 2 & if \quad x \leq -2 \\ -x^2 - 2x & if \quad x > -2 \end{cases}$



Approximate the relative extrema of each function.

5.



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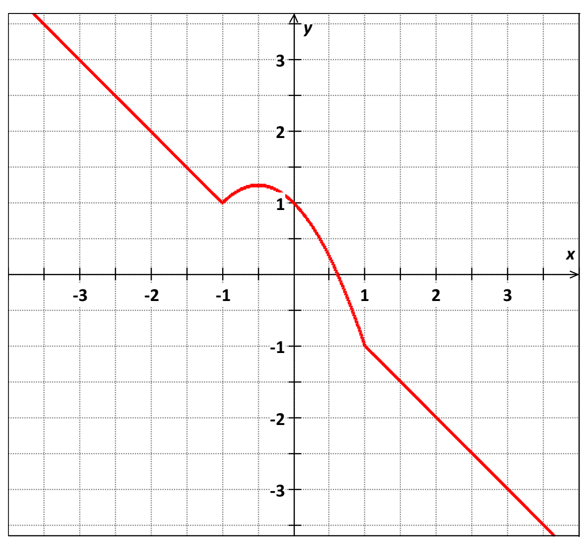
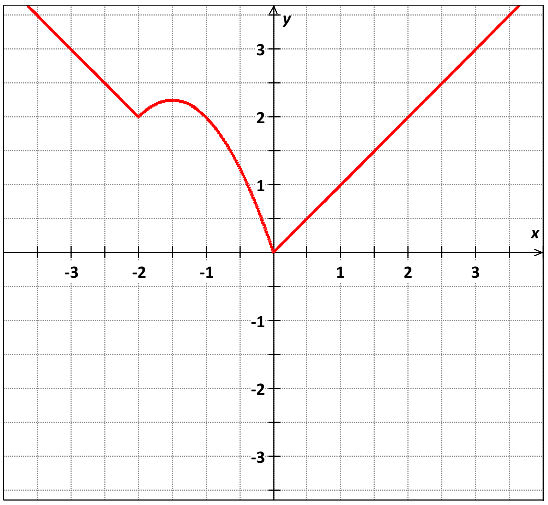


Name: ____ Period: __________ Date: _______________

Extrema and Average Rates of Change Assignment

Approximate the relative and absolute extrema of each function.

7.



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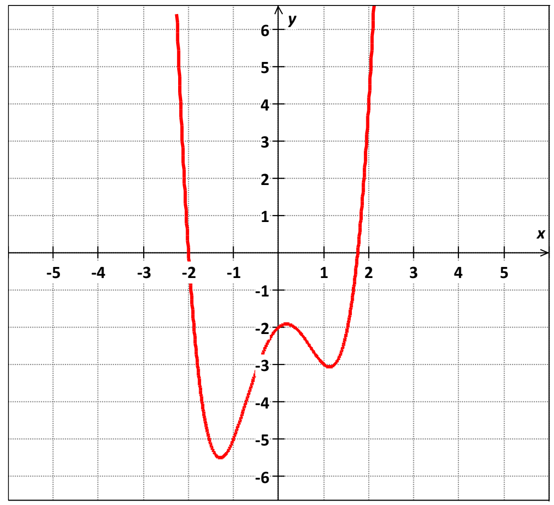
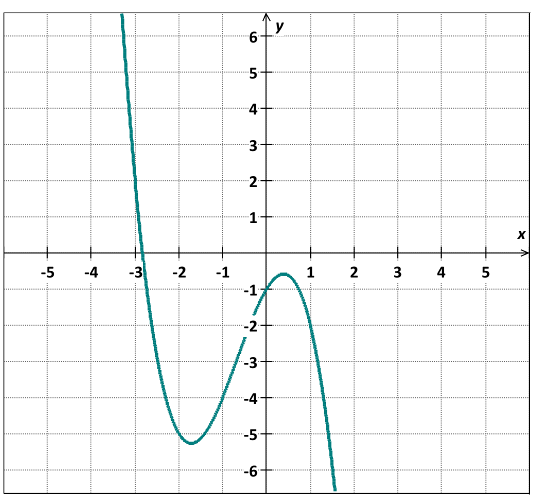


Estimate and classify the extrema for the graph of each function. Support the answers numerically.

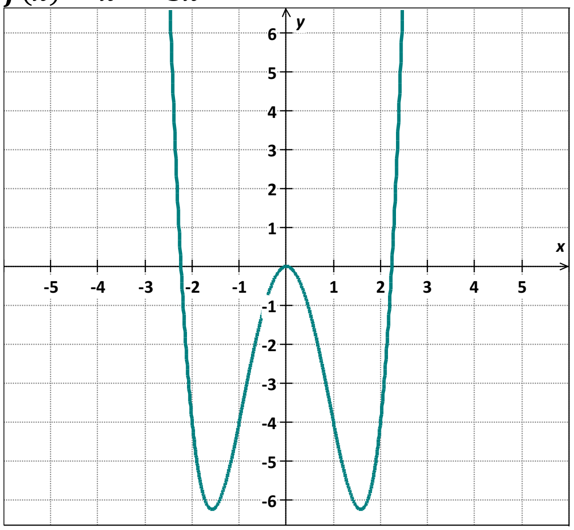
9. $f(x) = -x^3 - 2x^2 + 2x - 1$



Name: ____ Period: __________ Date: ______________

Extrema and Average Rates of Change Assignment

10. $f(x) = x^4 - 5x^2$



Find the average rate of change of each function on the given interval.

11. $f(x) = x^2 - 3x + 4 \quad [0; 4]$

12. $f(x) = \dfrac{2x+1}{5-x} \quad [1; 4]$

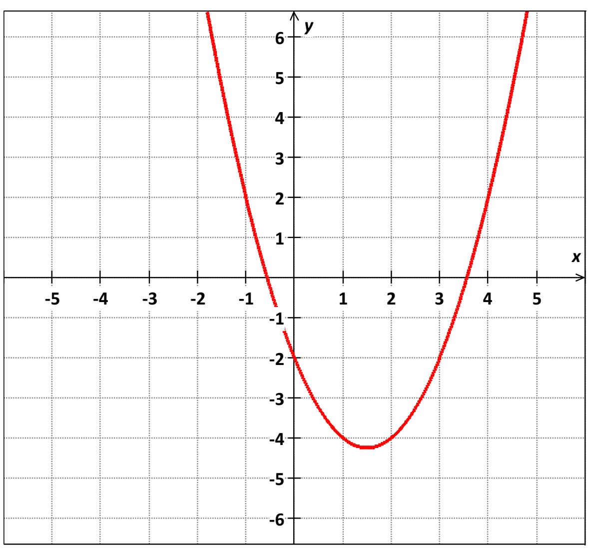
Name: ____ Period: ___________ Date: ________________

Extrema and Average Rates of Change Assignment

13. $f(x) = \sqrt{x+2}$ $[2; 7]$

14. $f(x) = \dfrac{x+2}{x}$ $[-4; 6]$

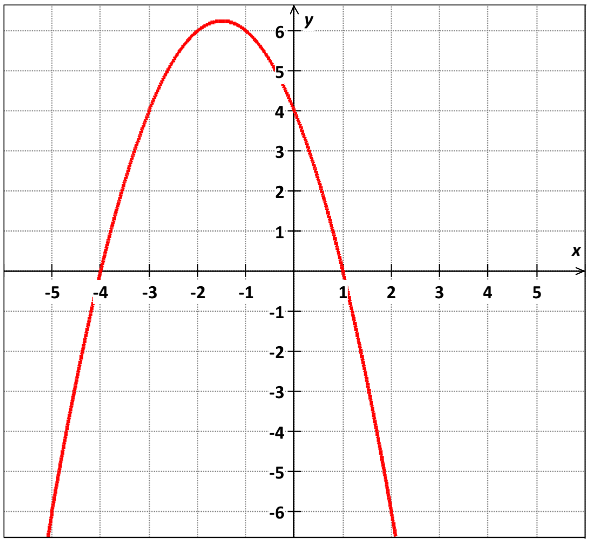
15. $f(x) = x^2 - 3x - 2$ $[0; 4]$



Name: ____ Period: __________ Date: _______________

Extrema and Average Rates of Change Assignment

16. $f(x) = -x^2 - 3x + 4 \quad [-3; 2]$



SOLVE THE PROBLEM

17. Maria records her distance from home over time. The values are shown in the table below. Find her average speed over the first 4 hours.

$t\ (hours)$	0	1	2	3	4	5	6	7
$D(t)(miles)$	0	60	110	180	220	360	420	600

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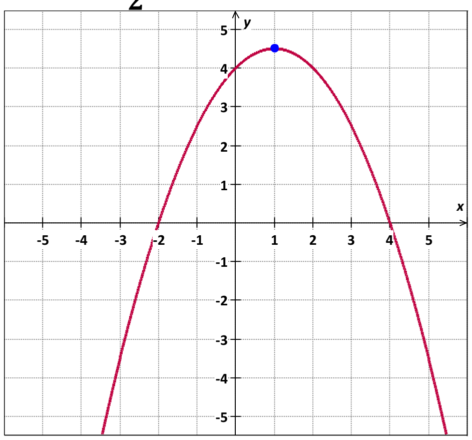
Name: ____ Period: ___________ Date: ________________

Extrema and Average Rates of Change Assignment

ANSWERS

Use the graph of each function to estimate intervals on which the function is increasing, decreasing, or constant. Support the answer numerically.

1. $f(x) = -\frac{1}{2}x^2 + x + 4$



From the graph, it appears that:

A function $-\frac{1}{2}x^2 + x + 4$ is increasing for $x < 1$

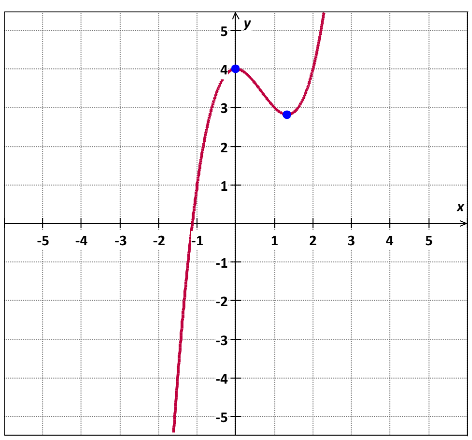
A function $-\frac{1}{2}x^2 + x + 4$ is decreasing for $x > 1$

The critical point is $(1; 4.5)$

x	-3	-2	-1	0	1	2	3
y	-3.5	0	2.5	4	4.5	4	2.5

The table supports this conjecture.

2. $f(x) = x^3 - 2x^2 + 4$



From the graph, it appears that:

A function $x^3 - 2x + 4$ is increasing for $x < 0$ and $x > 1.3$

A function $x^3 - 2x + 4$ is decreasing for $0 < x < 1.3$

The critical points are $(0; 4)$ and $(1.3; 2.8)$

$x < 0$

x	-4	-3	-2	-1
y	-92	-41	-14	1

$0 < x < 1.3$

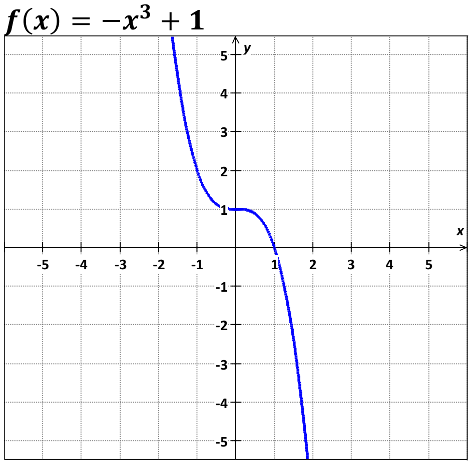
x	0.1	0.5	1	1.1
y	3.98	3.63	3	2.9

$x > 1.3$

x	1.5	2	3	4
y	2.87	4	13	36

The tables support this conjecture.

3. $f(x) = -x^3 + 1$



From the graph, it appears that:

A function $x^3 + 1$ is decreasing for $(-\infty; \infty)$

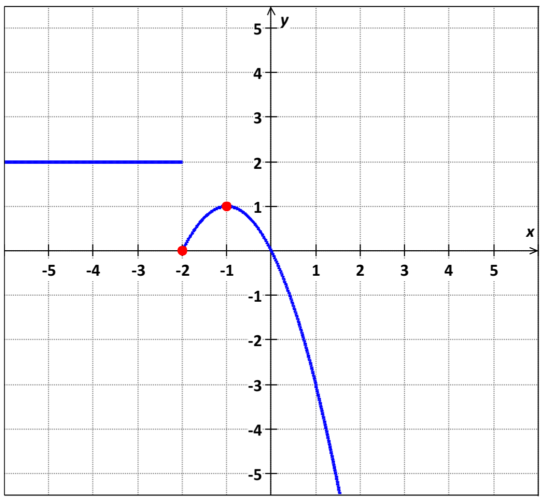
x	-3	-2	-1	0	1	2	3
y	28	9	2	1	0	-7	-26

The table supports this conjecture.

Name: ____ Period: __________ Date: _______________

Extrema and Average Rates of Change Assignment

4. $f(x) = \begin{cases} 2 & \textit{if} \quad x \le -2 \\ -x^2 - 2x & \textit{if} \quad x > -2 \end{cases}$



From the graph, it appears that:

A function $f(x)$ is constant for $x < -2$
A function $f(x)$ is increasing for $-2 < x < -1$
A function $f(x)$ is decreasing for $x > -1$

The critical points are $(-2; 0)$ *and* $(-1; 1)$

$x < -2$ *or* $(-\infty; -2)$

x	-5	-4	-3	-2
y	2	2	2	2

$-2 < x < -1$ *or* $(-2; -1)$

x	-1.9	-1.6	-1.3	-1.1
y	0.19	0.64	0.91	0.99

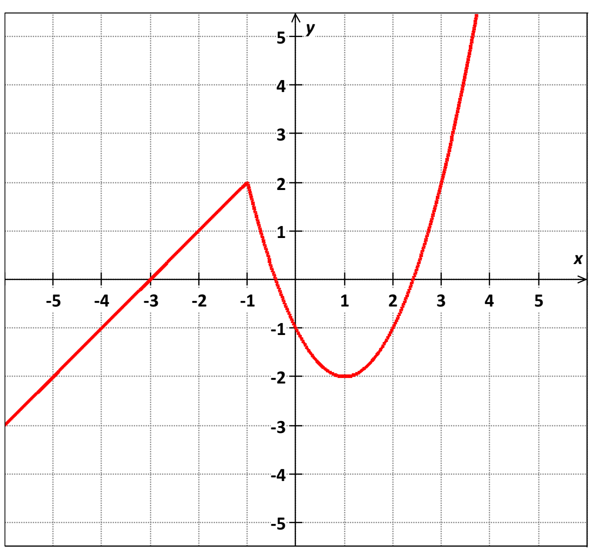
$x > -1$ *or* $(-1; \infty)$

x	0	1	2	3
y	0	-3	-8	-15

The tables support this conjecture.

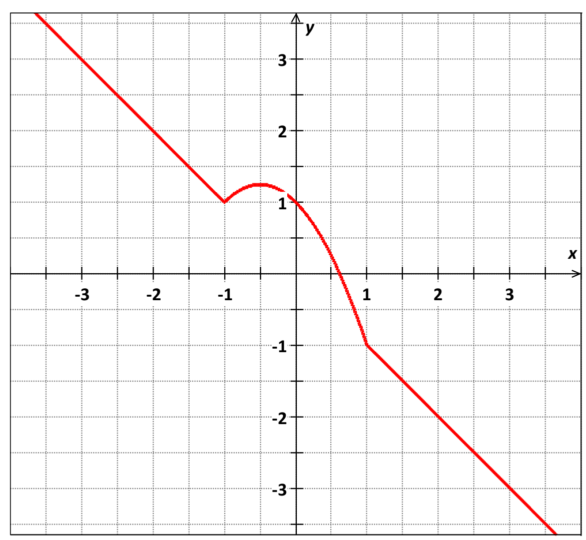
Approximate the relative extrema of each function.

5.



Relative minimum $(1; -2)$
Relative maximum $(-1; 2)$

6.



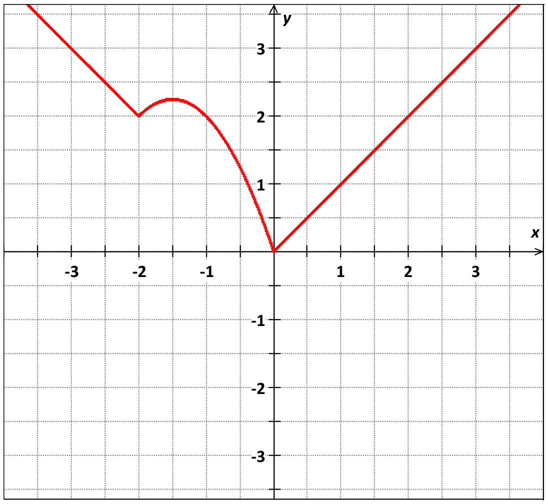
Relative minimum $(-1; 1)$
Relative maximum $(-0.5; 1.3)$

Name: ____ Period: __________ Date: ______________

Extrema and Average Rates of Change Assignment

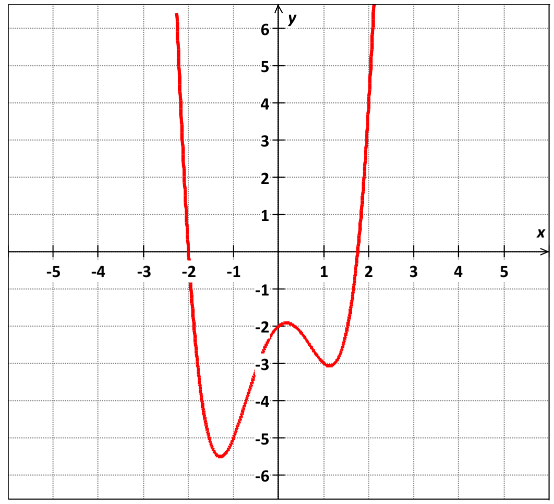
Approximate the relative and absolute extrema of each function.

7.



Relative minimum $(-2; 2)$
Relative maximum $(-1.5; 2.3)$
Absolute minimum $(0; 0)$
No absolute maxima

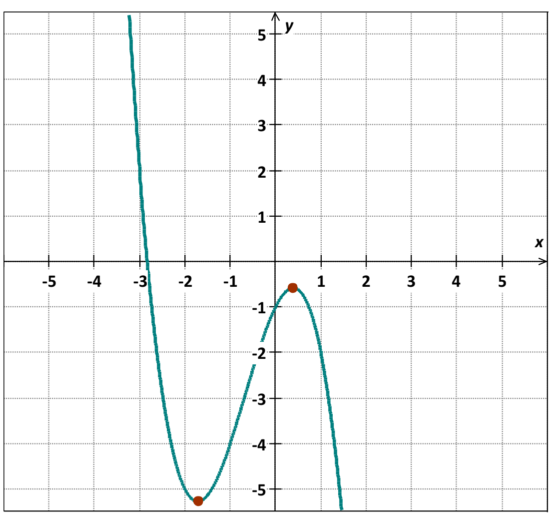
8.



Relative minimum $(1.1; -3)$
Relative maximum $(0.1; -1.9)$
Absolute minimum $(-1.3; -5.5)$
No absolute maxima

Estimate and classify the extrema for the graph of each function. Support the answers numerically.

9. $f(x) = -x^3 - 2x^2 + 2x - 1$



For interval $(-\infty, -1.72)$ the function is decreasing.
In $x = -1.72$, $f(x)$ has relative minimum.
For interval $(-1.72; 0.38)$ the function is increasing.
In $x = 0.38$, $f(x)$ has relative maximum.
For interval $(0.38; \infty)$ the function is decreasing.

From the graph, it appears that:
$f(x)$ has relative minimum in $x = -1.72$
$f(x)$ has relative maximum in $x = 0.38$

$\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$
$f(x)$ has no absolute maxima and absolute minima.

$x < -1.72$ or $(-\infty, -1.72)$

x	-4	-3.5	-3	-2.4	-2
y	87	22.63	3.5	-3.49	-5.1

$-1.72 < x < 0.38$ or $(-1.72; 0.38)$

x	-1.7	-1.5	-1.2	-1	0
	-5.3	-5.14	-4.55	-4	-1

$x > 0.38$ or $(0.38; \infty)$

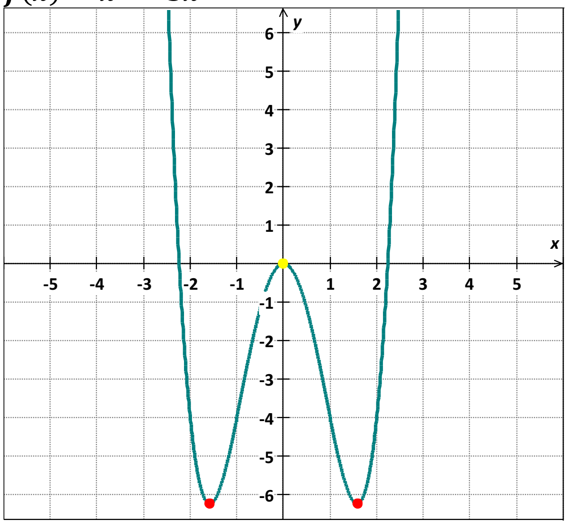
x	1	2	3	4	5
	-2	-13	-40	-89	-166

The tables support this conjecture.

Name: ____ Period: __________ Date: ______________

Extrema and Average Rates of Change Assignment

10. $f(x) = x^4 - 5x^2$



From the graph, it appears that:

$f(x)$ has relative maximum in $x = 0$

$f(x)$ has absolute minimum in $x = -1.58$ and $x = 1.58$

$\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

$f(x)$ has no absolute maxima.

$x < -1.58$ *or* $(-\infty; -1.58)$

x	-3	-2.5	-2	-1.52
y	36	7.81	-4	-6.21

$-1.58 < x < 0$ *or* $(-1.58; 0)$

x	-1.36	-1.20	-1.04	-0.88
	-5.83	-5.14	-4.24	-3.27

$0 < x < 1.58$ *or* $(0; 1.58)$

x	0.56	0.72	1.04	1.20
	-1.47	-2.32	-4.24	-5.14

$x > 1.58$ *or* $(1.58; \infty)$

x	1.68	2.0	2.48	2.98
y	-6.15	-4	7.08	32.95

The tables support this conjecture.

For interval $(-\infty; -1.58)$ the function is decreasing.
In $x = -1.58$, $f(x)$ has absolute minimum.
For interval $(-1.58; 0)$ the function is increasing.
In $x = 0$, $f(x)$ has relative maximum.
For interval $(0; 1.58)$ the function is decreasing.
In $x = 1.58$, $f(x)$ has absolute minimum.
For interval $(1.58; \infty)$ the function is increasing.

Find the average rate of change of each function on the given interval.

11. $f(x) = x^2 - 3x + 4 \quad [0; 4]$

$f(x) = x^2 - 3x + 4 \quad [0; 4]$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(0)}{4 - 0} =$$

$$= \frac{4^2 - 3 * 4 + 4 - (0^2 - 3 * 0 + 4)}{4} =$$

$$= \frac{16 - 12 + 4 - (0 + 0 + 4)}{4} =$$

$$= \frac{4 + 4 - 4}{4}$$

$$= \frac{4}{4} =$$

$$= 1$$

The average rate of change on the interval $[0; 4]$ is 1.

12. $f(x) = \frac{2x + 1}{5 - x} \quad [1; 4]$

$f(x) = \frac{2x + 1}{5 - x} \quad [1; 4]$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(1)}{4 - 1} =$$

$$= \frac{\frac{2 * 4 + 1}{5 - 4} - \frac{2 * 1 + 1}{5 - 1}}{3} =$$

$$= \frac{\frac{8 + 1}{1} - \frac{2 + 1}{4}}{3} =$$

$$= \frac{\frac{9}{1} - \frac{3}{4}}{3} =$$

$$= \frac{33}{12}$$

The average rate of change on the interval $[1; 4]$ is $\frac{33}{12}$

Name: ____ Period: __________ Date: _______________

Extrema and Average Rates of Change Assignment

13. $f(x) = \sqrt{x+2} \quad [2; 7]$

$f(x) = \sqrt{x+2} \quad [2; 7]$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(7) - f(2)}{7 - 2} =$$

$$= \frac{\sqrt{7+2} - (\sqrt{2+2})}{5} =$$

$$= \frac{\sqrt{9} - (\sqrt{4})}{5} =$$

$$= \frac{3 - 2}{5}$$

$$= \frac{1}{5}$$

The average rate of change on the interval $[2; 7]$ is $\frac{1}{5}$.

14. $f(x) = \frac{x+2}{x} \quad [-4; 6]$

$f(x) = \frac{x+2}{x} \quad [-4; 6]$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(6) - f(-4)}{6 - (-4)} =$$

$$= \frac{\frac{6+2}{6} - \frac{-4+2}{-4}}{10} =$$

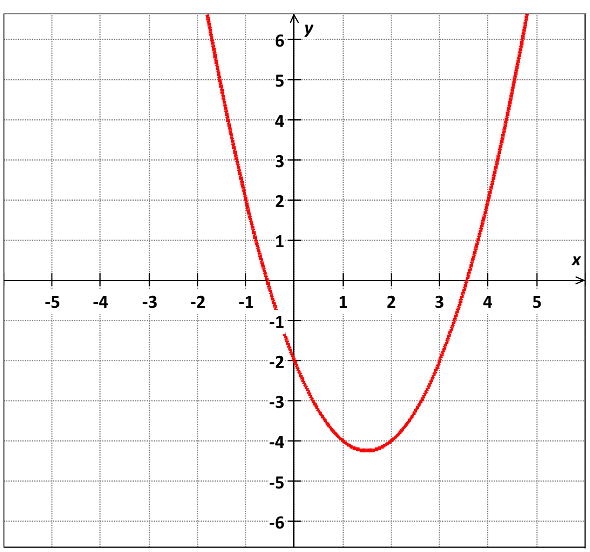
$$= \frac{\frac{8}{6} - \frac{2}{4}}{10} =$$

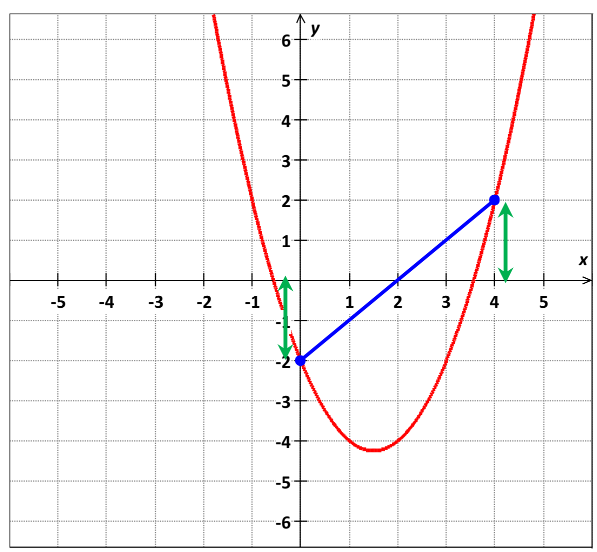
$$= \frac{\frac{10}{12}}{10} =$$

$$= \frac{1}{12}$$

The average rate of change on the interval $[-4; 6]$ is $\frac{1}{12}$

15. $f(x) = x^2 - 3x - 2 \quad [0; 4]$





$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(0)}{4 - 0} = \frac{2 - (-2)}{4} = \frac{4}{4} = 1$$

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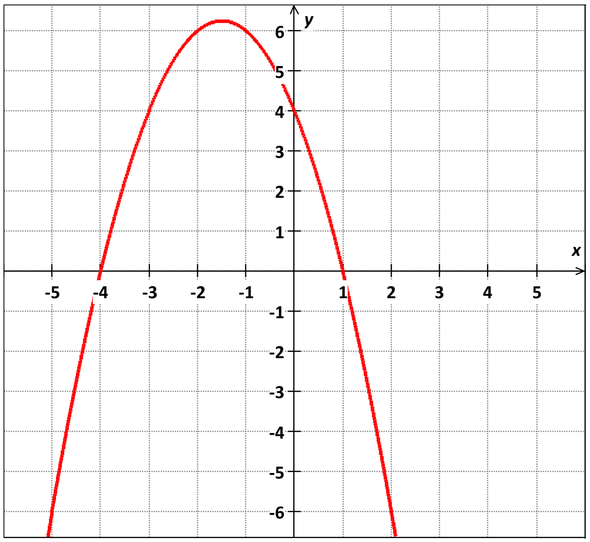
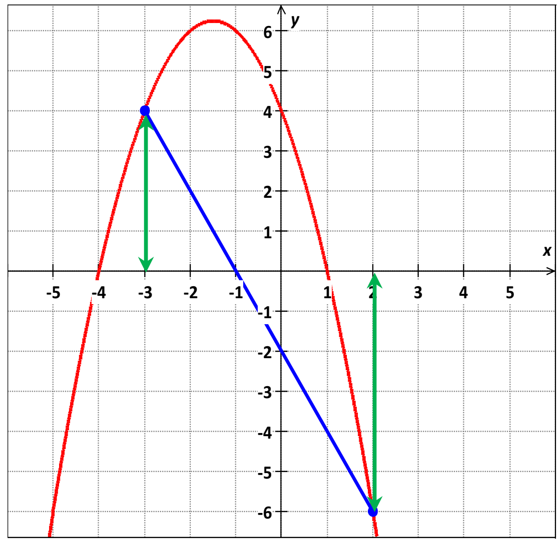
Name: ____ Period: ___________ Date: ________________

Extrema and Average Rates of Change Assignment

16. $f(x) = -x^2 - 3x + 4 \quad [-3; 2]$





$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(-3)}{2 - (-3)} = \frac{-6 - 4}{5} = \frac{-10}{5} = -2$$

SOLVE THE PROBLEM

17. Maria records her distance from home over time. The values are shown in the table below. Find her average speed over the first 4 hours.

t (hours)	0	1	2	3	4	5	6	7
$D(t)$(miles)	0	60	110	180	220	360	420	600

Average speed over the first 4 hours $= \frac{220-0}{4-0} = \frac{220}{4} = 55$

The average speed is **55 miles per hour**.