**Use the graph of each function to estimate intervals on which the function is increasing, decreasing, or constant. Support the answer numerically.**

|  |  |  |
| --- | --- | --- |
| **1.** | $$f\left(x\right)=-\frac{1}{2}x^{2}+x+4 $$ |  |
| **2.** | $$f\left(x\right)=x^{3}-2x^{2}+4$$ |  |
| **3.** | $$f\left(x\right)=-x^{3}+1$$ |  |
| **4.** | $$f\left(x\right)=\left\{\begin{array}{c}2 if x\leq -2\\-x^{2}-2x if x>-2\end{array}\right.$$ |  |

**Approximate the relative extrema of each function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **5.** |  | **6.** |  |
|  |  |  |  |

**Approximate the relative and absolute extrema of each function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **7.** |  | **8.** |  |
|  |  |  |  |

**Estimate and classify the extrema for the graph of each function. Support the answers numerically.**

|  |  |  |
| --- | --- | --- |
| **9.**  | $f\left(x\right)=-x^{3}-2x^{2}+2x-1 $ |  |
| **10.**  | $$f\left(x\right)=x^{4}-5x^{2}$$ |  |

**Find the average rate of change of each function on the given interval.**

|  |  |  |  |
| --- | --- | --- | --- |
| **11.**  | $f\left(x\right)=x^{2}-3x+4 \left[0;4\right]$ | **12.** | $$f\left(x\right)=\frac{2x+1}{5-x} \left[1;4\right]$$ |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **13.**  | $f\left(x\right)=\sqrt{x+2} \left[2;7\right]$ | **14.** | $$f\left(x\right)=\frac{x+2}{x} \left[-4;6\right]$$ |
|  |  |  |  |

|  |  |  |
| --- | --- | --- |
| **15.**  | $f\left(x\right)=x^{2}-3x-2 \left[0;4\right]$ |  |

|  |  |  |
| --- | --- | --- |
| **16.**  | $$f\left(x\right)=-x^{2}-3x+4 \left[-3;2\right]$$ |  |

**SOLVE THE PROBLEM**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **17.** | Maria records her distance from home over time. The values are shown in the table below. Find her average speed over the first 4 hours.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$t (hours)$$ | $$0$$ | $$1$$ | $$2$$ | $$3$$ | $$4$$ | $$5$$ | $$6$$ | $$7$$ |
| $$D\left(t\right)(miles)$$ | $$0$$ | $$60$$ | $$110$$ | $$180$$ | $$220$$ | $$360$$ | $$420$$ | $$600$$ |

 |
|  |  |

**ANSWERS**

**Use the graph of each function to estimate intervals on which the function is increasing, decreasing, or constant. Support the answer numerically.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1.** | $$f\left(x\right)=-\frac{1}{2}x^{2}+x+4 $$ | From the graph, it appears that:A function- $\frac{1}{2}x^{2}+x+4 $is increasing for$ x<1$A function -$\frac{1}{2}x^{2}+x+4 $is decreasing for$ x>1$The critical point is $(1;4.5)$

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$x$$ | $$-3$$ | $$-2$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ | $$3$$ |
| $$y$$ | $$-3.5$$ | $$0$$ | $$2.5$$ | $$4$$ | $$4.5$$ | $$4$$ | $$2.5$$ |

The table supports this conjecture**.** |
| **2.** | $$f\left(x\right)=x^{3}-2x^{2}+4$$ | From the graph, it appears that:A function $x^{3}-2x+4 $ is increasing for $ x<0 and x>1.3$A function $x^{3}-2x+4 $ is decreasing for $0<x<1.3$The critical points are $\left(0;4\right) and \left(1.3;2.8\right)$$$x<0 $$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-4$$ | $$-3$$ | $$-2$$ | $$-1$$ |
| $$y$$ | $$-92$$ | $$-41$$ | $$-14$$ | $$1$$ |

$$0<x<1.3$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0.1$$ | $$0.5$$ | $$1$$ | $$1.1$$ |
| $$y$$ | $$3.98$$ | $$3.63$$ | $$3$$ | $$2.9$$ |

$$x>1.3$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$1.5$$ | $$2$$ | $$3$$ | $$4$$ |
| $$y$$ | $$2.87$$ | $$4$$ | $$13$$ | $$36$$ |

The tables support this conjecture. |
| **3.** | $$f\left(x\right)=-x^{3}+1$$ | From the graph, it appears that:A function $x^{3}+1$ is decreasing for $ (-\infty ;\infty )$

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$x$$ | $$-3$$ | $$-2$$ | $$-1$$ | $$0$$ | $$1$$ | $$2$$ | $$3$$ |
| $$y$$ | $$28$$ | $$9$$ | $$2$$ | $$1$$ | $$0$$ | $$-7$$ | $$-26$$ |

The table supports this conjecture. |
| **4.** | $$f\left(x\right)=\left\{\begin{array}{c}2 if x\leq -2\\-x^{2}-2x if x>-2\end{array}\right.$$ | From the graph, it appears that:A function $f\left(x\right)$ is constant for $ x<-2 $A function $f\left(x\right)$ is increasing for $-2<x<-1$A function $f\left(x\right)$ is decreasing for $x>-1$The critical points are $\left(-2;0\right) and \left(-1;1\right)$$$x<-2 or \left(-\infty ;-2\right) $$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-5$$ | $$-4$$ | $$-3$$ | $$-2$$ |
| $$y$$ | $$2$$ | $$2$$ | $$2$$ | $$2$$ |

$$-2<x<-1 or \left(-2;-1\right)$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-1.9$$ | $$-1.6$$ | $$-1.3$$ | $$-1.1$$ |
| $$y$$ | $$0.19$$ | $$0.64$$ | $$0.91$$ | $$0.99$$ |

$$x>-1 or \left(-1;\infty \right)$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0$$ | $$1$$ | $$2$$ | $$3$$ |
| $$y$$ | $$0$$ | $$-3$$ | $$-8$$ | $$-15$$ |

The tables support this conjecture. |

**Approximate the relative extrema of each function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **5.** |  | **6.** |  |
|  | Relative minimum $\left(1;-2\right)$Relative maximum$ \left(-1;2\right)$ |  | Relative minimum $\left(-1;1\right)$Relative maximum$ \left(-0.5;1.3\right)$ |

**Approximate the relative and absolute extrema of each function.**

|  |  |  |  |
| --- | --- | --- | --- |
| **7.** |  | **8.** |  |
|  | Relative minimum $\left(-2;2\right)$Relative maximum$ \left(–1.5;2.3\right)$Absolute minimum $\left(0;0\right)$No absolute maxima |  | Relative minimum $\left(1.1;-3\right)$Relative maximum$ \left(0.1;-1.9\right)$Absolute minimum $\left(-1.3;-5.5\right)$No absolute maxima |

**Estimate and classify the extrema for the graph of each function. Support the answers numerically.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **9.**  | $f\left(x\right)=-x^{3}-2x^{2}+2x-1 $For interval $\left(-\infty ,-1.72\right)$the function is decreasing.In $ x=-1.72, $$ f\left(x\right)$has relative minimum.For interval $\left(-1.72;0.38\right)$the function is increasing.In $ x=0.38, $$ f\left(x\right)$has relative maximum.For interval $\left(0.38;\infty \right)$the function is decreasing. | From the graph, it appears that:$f\left(x\right)$has relative minimum in $ x=-1.72$$f\left(x\right)$has relative maximum in $ x=0.38$$\lim\_{x\to -\infty }f\left(x\right)=\infty $and$\lim\_{x\to \infty }f\left(x\right)=-\infty $$f\left(x\right)$has no absolute maxima and absolute minima.$$x<-1.72 or \left(-\infty ,-1.72\right) $$

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$x$$ | $$-4$$ | $$-3.5$$ | $$-3$$ | $$-2.4$$ | $$-2$$ |
| $$y$$ | $$87$$ | $$22.63$$ | $$3.5$$ | $$-3.49$$ | $$-5.1$$ |

$$-1.72<x<0.38 or \left(-1.72;0.38\right)$$

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$x$$ | $$-1.7$$ | $$-1.5$$ | $$-1.2$$ | $$-1$$ | $$0$$ |
|  | $$-5.3$$ | $$-5.14$$ | $$-4.55$$ | $$-4$$ | $$-1$$ |

$$x>0.38 or \left(0.38;\infty \right) $$

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$x$$ | $$1$$ | $$2$$ | $$3$$ | $$4$$ | $$5$$ |
|  | $$-2$$ | $$-13$$ | $$-40$$ | $$-89$$ | $$-166$$ |

The tables support this conjecture. |
| **10.**  | $$f\left(x\right)=x^{4}-5x^{2}$$For interval $\left(-\infty ;-1.58 \right)$the function is decreasing.In $ x=-1.58 , $$ f\left(x\right)$ has absolute minimum.For interval $\left(-1.58 ;0\right)$the function is increasing.In $ x=0, $$ f\left(x\right)$hasrelative maximum.For interval $\left(0;1.58 \right)$the function is decreasing.In $ x=1.58 , $$ f\left(x\right)$ hasabsolute minimum.For interval $\left(1.58 ;\infty \right)$the function is increasing. | From the graph, it appears that:$f\left(x\right)$has relative maximum in $ x=0$$f\left(x\right)$has absolute minimum in $ x=-1.58$and $ x=1.58$$\lim\_{x\to -\infty }f\left(x\right)=\infty $and$\lim\_{x\to \infty }f\left(x\right)=\infty $$f\left(x\right)$has no absolute maxima.$$x<-1.58 or \left(-\infty ;-1.58 \right) $$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-3$$ | $$-2.5$$ | $$-2$$ | $$-1.52$$ |
| $$y$$ | $$36$$ | $$7.81$$ | $$-4$$ | $$-6.21$$ |

$$-1.58 <x<0 or \left(-1.58 ;0\right) $$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$-1.36$$ | $$-1.20$$ | $$-1.04$$ | $$-0.88$$ |
|  | $$-5.83$$ | $$-5.14$$ | $$-4.24$$ | $$-3.27$$ |

$$0<x<1.58 or \left(0;1.58 \right) $$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$0.56$$ | $$0.72$$ | $$1.04$$ | $$1.20$$ |
|  | $$-1.47$$ | $$-2.32$$ | $$-4.24$$ | $$-5.14$$ |

$$x>1.58 or\left(1.58 ;\infty \right) $$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$1.68$$ | $$2.0$$ | $$2.48$$ | $$2.98$$ |
| $$y$$ | $$-6.15$$ | $$-4$$ | $$7.08$$ | $$32.95$$ |

The tables support this conjecture. |

**Find the average rate of change of each function on the given interval.**

|  |  |  |  |
| --- | --- | --- | --- |
| **11.**  | $f\left(x\right)=x^{2}-3x+4 \left[0;4\right]$ | **12.** | $$f\left(x\right)=\frac{2x+1}{5-x} \left[1;4\right]$$ |
|  | $$f\left(x\right)=x^{2}-3x+4 \left[0;4\right]$$$$\frac{f\left(x\_{2}\right)-f\left(x\_{1}\right)}{x\_{2}-x\_{1}}=\frac{f\left(4\right)-f\left(0\right)}{4-0}=$$$$=\frac{4^{2}-3\*4+4-(0^{2}-3\*0+4)}{4}=$$$$=\frac{16-12+4 -\left(0+0+4\right)}{4}=$$$$=\frac{4+4 -4}{4}$$$$=\frac{4}{4}=$$$$=1$$The average rate of change on the interval$ \left[0;4\right]$is $ 1$ . |  | $$f\left(x\right)=\frac{2x+1}{5-x} \left[1;4\right] $$$$\frac{f\left(x\_{2}\right)-f\left(x\_{1}\right)}{x\_{2}-x\_{1}}=\frac{f\left(4\right)-f\left(1\right)}{4-1}=$$$$=\frac{\frac{2\*4+1}{5-4}-\frac{2\*1+1}{5-1}}{3}=$$$$=\frac{\frac{8+1}{1}-\frac{2+1}{4}}{3}=$$$$=\frac{\frac{9}{1}-\frac{3}{4}}{3}=$$$$=\frac{33}{12}$$The average rate of change on the interval$ \left[1;4\right]$is $ \frac{33}{12}$ |

|  |  |  |  |
| --- | --- | --- | --- |
| **13.**  | $f\left(x\right)=\sqrt{x+2} \left[2;7\right]$ | **14.** | $$f\left(x\right)=\frac{x+2}{x} \left[-4;6\right]$$ |
|  | $$f\left(x\right)=\sqrt{x+2} \left[2;7\right]$$$$\frac{f\left(x\_{2}\right)-f\left(x\_{1}\right)}{x\_{2}-x\_{1}}=\frac{f\left(7\right)-f\left(2\right)}{7-2}=$$$$=\frac{\sqrt{7+2}-(\sqrt{2+2})}{5}=$$$$=\frac{\sqrt{9}-(\sqrt{4})}{5}=$$$$=\frac{3-2}{5}$$$$=\frac{1}{5}$$The average rate of change on the interval$ \left[2;7\right]$is $ \frac{1}{5}$ . |  | $$f\left(x\right)=\frac{x+2}{x} \left[-4;6\right] $$$$\frac{f\left(x\_{2}\right)-f\left(x\_{1}\right)}{x\_{2}-x\_{1}}=\frac{f\left(6\right)-f\left(-4\right)}{6-(-4)}=$$$$=\frac{\frac{6+2}{6}-\frac{-4+2}{-4}}{10}=$$$$=\frac{\frac{8}{6}-\frac{2}{4}}{10}=$$$$=\frac{\frac{10}{12}}{10}=$$$$=\frac{1}{12}$$The average rate of change on the interval$ \left[-4;6\right]$is $ \frac{1}{12}$ |

|  |  |  |
| --- | --- | --- |
| **15.**  | $f\left(x\right)=x^{2}-3x-2 \left[0;4\right]$ | $$\frac{f\left(x\_{2}\right)-f\left(x\_{1}\right)}{x\_{2}-x\_{1}}=\frac{f\left(4\right)-f\left(0\right)}{4-0}=\frac{2-(-2)}{4}=\frac{4}{4}=1$$ |

|  |  |  |
| --- | --- | --- |
| **16.**  | $$f\left(x\right)=-x^{2}-3x+4 \left[-3;2\right]$$ | $$\frac{f\left(x\_{2}\right)-f\left(x\_{1}\right)}{x\_{2}-x\_{1}}=\frac{f\left(2\right)-f\left(-3\right)}{2-\left(-3\right)}=\frac{-6-4}{5}=\frac{-10}{5}=-2$$ |

**SOLVE THE PROBLEM**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **17.** | Maria records her distance from home over time. The values are shown in the table below. Find her average speed over the first 4 hours.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$t (hours)$$ | $$0$$ | $$1$$ | $$2$$ | $$3$$ | $$4$$ | $$5$$ | $$6$$ | $$7$$ |
| $$D\left(t\right)(miles)$$ | $$0$$ | $$60$$ | $$110$$ | $$180$$ | $$220$$ | $$360$$ | $$420$$ | $$600$$ |

 |
|  | Average speed over the first 4 hours $=\frac{220-0}{4-0}=\frac{220}{4}=55$The average speed is **55 miles per hour.** |