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Continuity, End Behavior, and Limits Guided Notes

The graph of **a continuous function** has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.

One condition for a function $f(x)$ to be continuous at $x = c$ is that the function must approach a unique function value as x -values approach c from the left and right sides. The concept of approaching a value without necessarily ever reaching it is called **a limit.**

If the value of $f(x)$approaches a unique value L as x approaches c from each side, then the limit of $f(x)$ as x approaches c is L. $\lim_{x\to c} f(x) = L$

Functions that are not continuous are discontinuous. Graphs that are discontinuous can exhibit:

- Infinite discontinuity (A function has an infinite discontinuity at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right)
- Jump discontinuity,(A function has a jump discontinuity at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.
- Removable discontinuity, also called point discontinuity (function has a removable discontinuity if the function is continuous everywhere except for a hole at $x = c$.

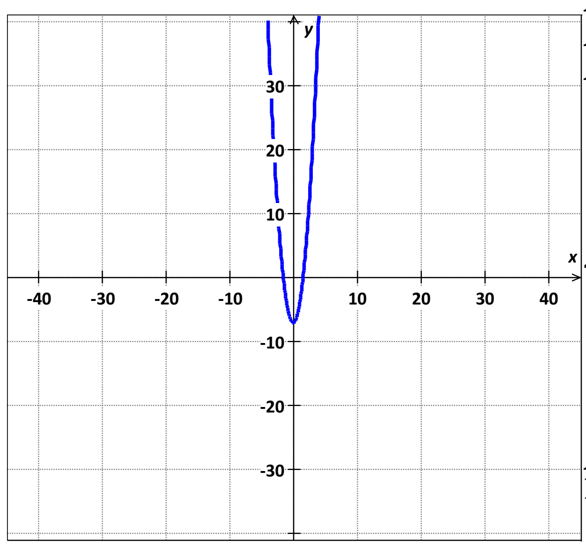
Continuity Test

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

1. $f(x)$ is defined at c. $f(c)$exists.
2. $f(x)$ approaches the same function value to the left and right of c. $\lim_{x\to c} f(x)\ exists$
3. The function value that $f(x)$approaches from each side of c is $f(c)$. $\lim_{x\to c} f(x) = f(c)$

Sample Problem 1: Determine whether each function is continuous at the given x -values. Justify using the continuity test. If discontinuous, identify the type of discontinuity.

a. $f(x) = 3x^2 + x - 7 \quad at \ x = 1$



$f(x) = 3x^2 + x - 7 \quad at \ x = 1$

$f(1) = 3 * 1^2 + 1 - 7$

$f(1) = -3$

$f(1)\ exists$

$x \to 1^- \quad y \to -3$

x	0.9	0.99	0.999
$f(x)$	-3.67	-3.0697	-3.006997

$x \to 1^+ \quad y \to -3$

x	1.1	1.01	1.001
$f(x)$	-2.27	-2.9297	-2.992997

$f(1) = -3\ and\ y \to -3\ from\ both\ side\ of\ x = 1$

$\lim_{x\to 1} 3x^2 + x - 7 = f(1)$

$f(x) = 3x^2 + x - 7\ is\ continuous\ at\ x = 1$

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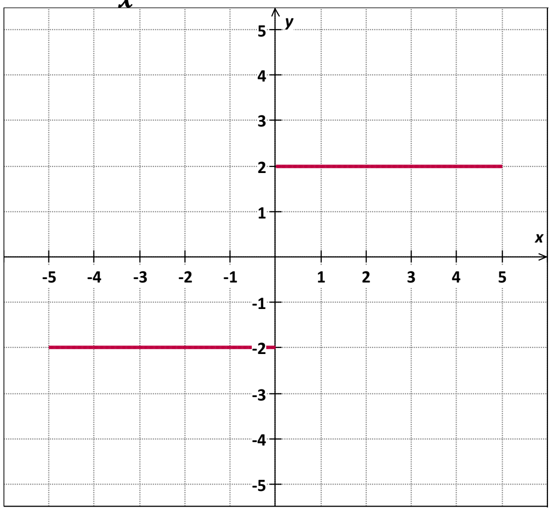
b. $f(x) = \frac{|2x|}{x} \quad at \quad x = 0$



$f(x) = \frac{|2x|}{x} \quad at \quad x = 0$

$f(0) = \frac{|2 * 0|}{0} = \frac{0}{0}$

$The\ function\ is\ undefined\ at\ x = 0$

$x \to 0^- \quad y \to -2$

x	-0.1	-0.01	-0.001
$f(x)$	-2	-2	-2

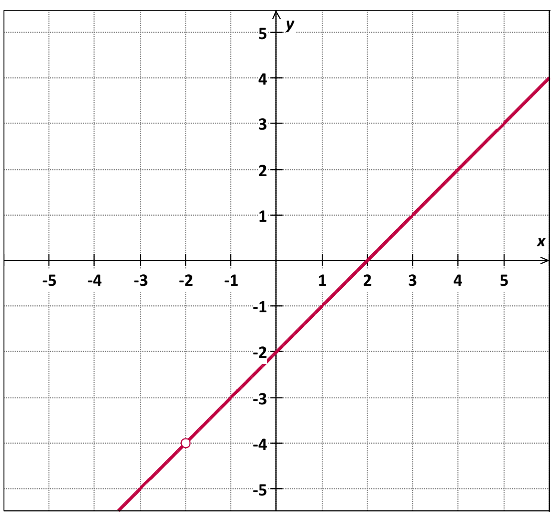
$x \to 0^+ \quad y \to 2$

x	0.1	0.01	0.001
$f(x)$	2	2	2

$f(x) = \frac{|2x|}{x}$ *has* ***jump discontinuity*** *at* $x = 0$

$since\ y\ values\ are\ 2\ and -2\ on\ opposite\ sides\ of\ x = 0$

c. $f(x) = \frac{x^2 - 4}{x + 2} \quad at \quad x = -2$



$f(x) = \frac{x^2 - 4}{x + 2} \quad at \quad x = -2$

$f(-2) = \frac{(-2)^2 - 4}{-2 + 2} = \frac{0}{0}$

$f(x)\ is\ undefined\ in\ x = -2$

$f(x) = \frac{x^2-4}{x+2}\ \ is\ discontinuous\ at\ x = -2$

$x \to -2^- \quad y \to -4$

x	-2.1	-2.01	-2.001
$f(x)$	-4.1	-4.01	-4.001

$x \to -2^+ \quad y \to -4$

x	-1.9	-1.99	-1.999
$f(x)$	-3.9	-3.99	-3.999

$f(x) = \frac{x^2 - 4}{x + 2}$ *has* ***point discontinuity*** *at* $x = -2$

$since\ \ y\ value\ is -4\ on\ opposite\ sides\ of\ x = -2$

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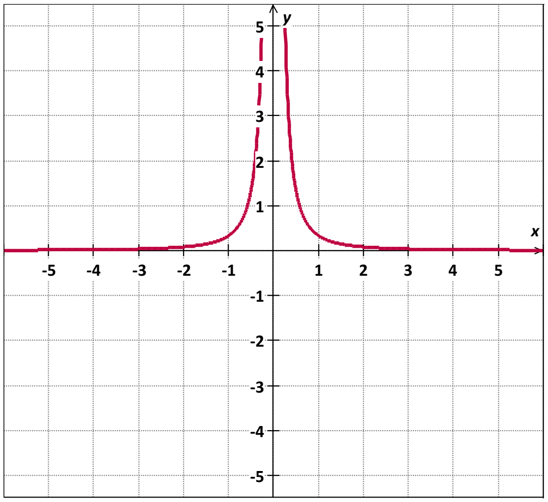
d. $f(x) = \dfrac{1}{3x^2} \quad at \quad x = 0$



$f(x) = \dfrac{1}{3x^2} \quad at \quad x = 0$

$f(0) = \dfrac{1}{3 * 0^2} = \infty$

$f(x)$ *is undefined in* $x = 0$

$f(x) = \dfrac{1}{3x^2}$ *is discontinuous at* $x = 0$

$x \to 0^- \quad y \to +\infty$

x	-0.1	-0.01	-0.001
$f(x)$	33.33	$3,333.33$	$333,333.33$

$x \to 0^+ y \to +\infty$

x	0.1	0.01	0.001
$f(x)$	33.33	$3,333.33$	$333,333.33$

$f(x) = \dfrac{1}{3x^2}$ *has infinity discontinuity at* $x = 0$ *since* y *value is* $+\infty$ *when* $x \to 0$

Intermediate Value Theorem

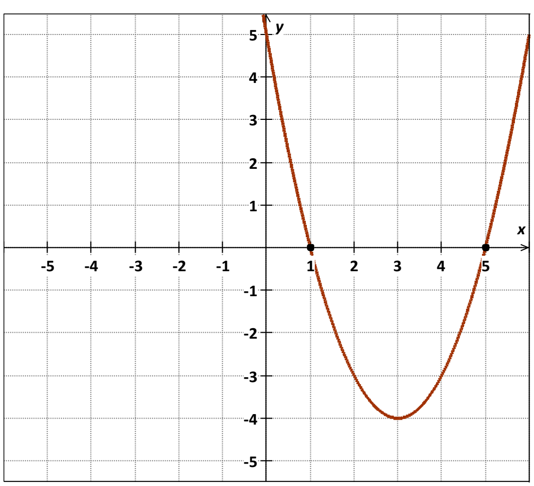
If $f(x)$is a continuous function and $a < b$ and there is a value n such that n is between $f(a)$ and $f(b)$, then there is a number$c$, such that $a < c < b$ and $f(c) = n$

The Location Principle

If $f(x)$ is a continuous function and $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value c, such that $a < c < b$ and $f(c) = 0$. That is, there is a zero between a and b.

Sample Problem 2: **Determine between which consecutive integers the real zeros of function are located on the given interval.**

a. $f(x) = (x - 3)^2 - 4 \qquad [0, 6]$



x	0	1	2	3	4	5	6
y	5	0	-3	-4	-3	0	5

$f(0)$ is positive and $f(2)$ is negative,
$f(x)$ *change sign in* $0 \le x \le 2$
$f(4)$ is negative and $f(6)$ is positive
$f(x)$ *change sign in* $4 \le x \le 6$

$f(x)$ has zeros in intervals:
$0 \le x \le 2 \quad and \quad 4 \le x \le 6$

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Continuity, End Behavior, and Limits Guided Notes

End Behavior

The end behavior of a function describes what the y -values do as $|x|$ becomes greater and greater.

When x becomes greater and greater, we say that x approaches infinity, and we write $x \to +\infty$.

When x becomes more and more negative, we say that x approaches negative infinity, and we write $x \to -\infty$.

The same notation can also be used with y or $f(x)$ and with real numbers instead of infinity.

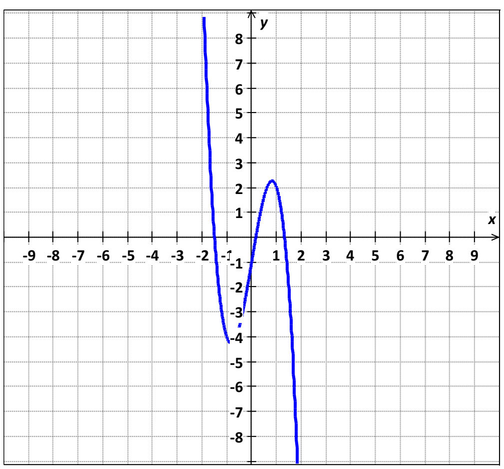
Left - End Behavior (as x becomes more and more negative): $\lim\limits_{x\to-\infty} f(x)$

Right - End Behavior (as x becomes more and more positive): $\lim\limits_{x\to+\infty} f(x)$

The $f(x)$ values may approach negative infinity, positive infinity, or a specific value.

Sample Problem 3: Use the graph of each function to describe its end behavior. Support the conjecture numerically.

a. $f(x) = -3x^3 + 6x - 1$

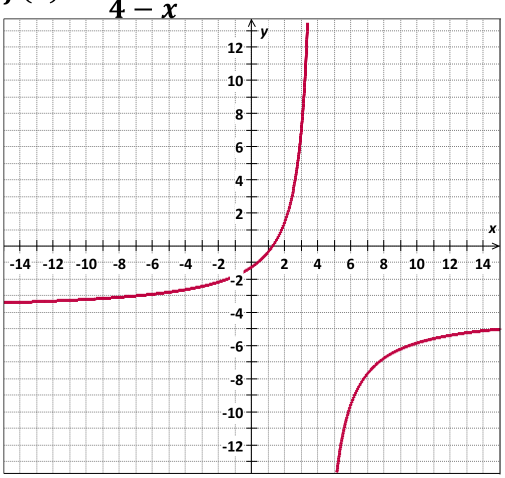


From the graph, it appears that:
$f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ – as $x \to \infty$

The table supports this conjecture.

x	-10^4	-10^3	0	10^3	10^4
y	$3 * 10^{12}$	$3 * 10^9$	-1	-$3 * 10^9$	-$3 * 10^{12}$

b. $f(x) = \dfrac{4x - 5}{4 - x}$



From the graph, it appears that:
$f(x) \to -4$ as $x \to -\infty$ and $f(x) \to -4$ as $x \to \infty$

The table supports this conjecture.

x	-10^4	-10^3	0	10^3	10^4
y	-3.9989	-3.9890	-1.25	-4.001	-4.0011

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Increasing, Decreasing, and Constant Functions

A function $\boldsymbol{f}$ is increasing on an interval $\boldsymbol{I}$ if and only if for every $\boldsymbol{a}$ and $\boldsymbol{b}$ contained in $\boldsymbol{I}$, $(\boldsymbol{a}) < \boldsymbol{f}(\boldsymbol{b})$, whenever $\boldsymbol{a} < \boldsymbol{b}$.

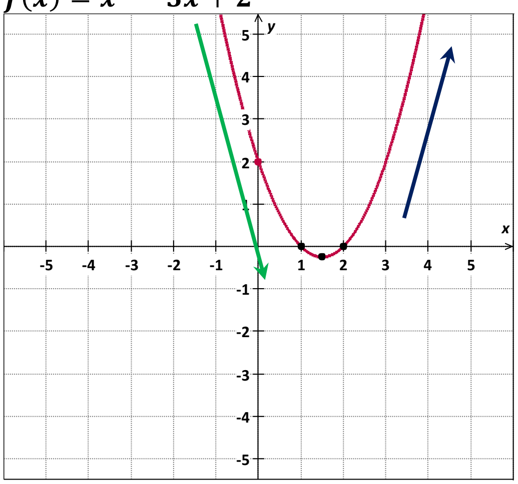
A function $\boldsymbol{f}$ is decreasing on an interval $\boldsymbol{I}$ if and only if for every $\boldsymbol{a}$ and $\boldsymbol{b}$ contained in $\boldsymbol{I}$, $\boldsymbol{f}(\boldsymbol{a}) > \boldsymbol{f}(\boldsymbol{b})$ whenever $\boldsymbol{a} < \boldsymbol{b}$.

A function $\boldsymbol{f}$ remains constant on an interval $\boldsymbol{I}$ if and only if for every $\boldsymbol{a}$ and $\boldsymbol{b}$ contained in $\boldsymbol{I}$, $\boldsymbol{f}(\boldsymbol{a}) = \boldsymbol{f}(\boldsymbol{b})$ whenever $\boldsymbol{a} < \boldsymbol{b}$.

Points in the domain of a function where the function changes from increasing to decreasing or from decreasing to increasing are called **critical points.**

Sample Problem 4: **Determine the interval(s) on which the function is increasing and the interval(s) on which the function is decreasing.**

a. $\boldsymbol{f(x) = x^2 - 3x + 2}$



From the graph, it appears that:

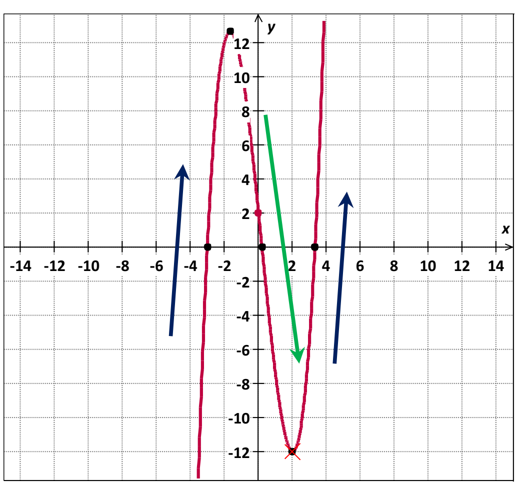
A function $\boldsymbol{x^2 - 3x + 2}$ is decreasing for $\boldsymbol{x < 1.5}$

A function $\boldsymbol{x^2 - 3x + 2}$ is increasing for $\boldsymbol{x > 1.5}$

The table supports this conjecture.

x	-1	0	1	1.5	2	3
y	6	2	0	-0.25	-5.5	2

b. $\boldsymbol{f(x) = x^3 - \frac{1}{2}x^2 - 10x + 2}$



From the graph, it appears that:

A function $\boldsymbol{x^3 - \frac{1}{2}x^2 - 10x + 2}$ is increasing:

$\boldsymbol{x < -1.66}$ ***and*** $\boldsymbol{x > 2}$

A function $\boldsymbol{x^3 - \frac{1}{2}x^2 - 10x + 2}$ is decreasing:

$\boldsymbol{-1.66 < x < 2}$

The table supports this conjecture.

x	-2	-1.66	-1	0	2	3
y	12	12.65	10.5	2	-12	-5.5