The graph of **a continuous function** has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.

One condition for a function$ f\left(x\right)$ to be continuous at $x=c$ is that the function must approach a unique function value as $x$ -values approach $c$ from the left and right sides. The concept of approaching a value without necessarily ever reaching it is called **a limit.**

If the value of $f\left(x\right)$approaches a unique value$ L$as $x$ approaches $c$ from each side, then the limit of $f\left(x\right) $as $x$ approaches $c$ is$L. \lim\_{x\to c }f\left(x\right)=L$

Functions that are not continuous are discontinuous. Graphs that are discontinuous can exhibit:

* Infinite discontinuity (A function has an infinite discontinuity at $x=c$ if the function value increases or decreases indefinitely as $x$ approaches $c$ from the left and right)
* Jump discontinuity,( A function has a jump discontinuity at $x=c$ if the limits of the function as $x$ approaches $c$ from the left and right exist but have two distinct values.
* Removable discontinuity, also called point discontinuity (function has a removable discontinuity if the function is continuous everywhere except for a hole at $x=c$.

 **Continuity Test**

A function$ f\left(x\right)$ is continuousat $x=c$if it satisfies the following conditions**.**

1. $f\left(x\right) $is defined at **c.** $f(c)$exists.
2. $f\left(x\right) $approaches the same function value to the left and right of$c.$$\lim\_{x\to c }f\left(x\right) exists$
3. The function value that$f\left(x\right)$approaches from each side of$c$is$f\left(c\right).$ $\lim\_{x\to c }f\left(x\right)=f(c)$

**Sample Problem 1: Determine whether each function is continuous at the given** $x$ **-values. Justify using the continuity test. If discontinuous, identify the type of discontinuity.**

|  |  |  |
| --- | --- | --- |
| **a.**  | $f\left(x\right)=3x^{2}+x-7 at x= 1$ | $$ $$ |
| **b.**  | $$f\left(x\right)=\frac{\left|2x\right|}{x} at x=0$$ |  |
| **c.**  | $$f\left(x\right)=\frac{x^{2}-4}{x+2} at x=-2$$ | $$ $$ |

|  |  |  |
| --- | --- | --- |
| **d.**  | $$f\left(x\right)=\frac{1}{3x^{2}} at x=0$$ | $$ $$ |

**Intermediate Value Theorem**

If $f\left(x\right)$is a continuous function and $a<b$ and there is a value $n$ such that $n$ is between $f\left(a\right)$ and $f\left(b\right) , $then there is a number$c$, such that $a<c<b$ and $f\left(c\right)=n$

**The Location Principle**

If $f\left(x\right)$ is a continuous function and$ f\left(a\right)$ and $f\left(b\right)$ have opposite signs, then there exists at least one value $c$, such that $a<c<b$ and $f\left(c\right)=0$. That is, there is a zero between $a$ and $b.$

**Sample Problem 2**: **Determine between which consecutive integers the real zeros of function are located on the given interval.**

|  |  |  |
| --- | --- | --- |
| **a.**  | $f\left(x\right)=\left(x-3\right)^{2}-4 [0,6] $ |  |

**End Behavior**

The end behavior of a function describes what the $y$ -values do as $\left|x\right|$ becomes greater and greater.

When $x$ becomes greater and greater, we say that $x$ approaches infinity, and we write$ x\rightarrow +\infty $.

When$ x$ becomes more and more negative, we say that $x$ approaches negative infinity, and we write$ x\rightarrow -\infty $.

The same notation can also be used with $y$ or $f\left(x\right)$ and with real numbers instead of infinity.

Left - End Behavior (as $x$ becomes more and more negative):$ \lim\_{x\to -\infty }f\left(x\right)$

Right - End Behavior (as $x$ becomes more and more positive): $\lim\_{x\to +\infty }f\left(x\right)$

The $f\left(x\right)$ values may approach negative infinity, positive infinity, or a specific value.

**Sample Problem 3**: **Use the graph of each function to describe its end behavior. Support the conjecture numerically.**

|  |  |  |
| --- | --- | --- |
| **a.**  | $f\left(x\right)=-3x^{3}+6x-1 $ |  |
| **b.**  | $$f\left(x\right)=\frac{4x-5}{4-x}$$ |  |

**Increasing, Decreasing, and Constant Functions**

A function $f$ is increasing on an interval $I$ if and only if for every $a$ and $b$ contained in $I$, $\left(a\right)<f\left(b\right)$ , whenever $a<b$ .

A function $f$ is decreasing on an interval $I$ if and only if for every $a$ and $b$ contained in $I$, $f\left(a\right)>f\left(b\right)$ whenever $a<b$ .

A function $f$ remains constant on an interval $I$ if and only if for every $a$ and $b$ contained in $I$, $f\left(a\right)=f\left(b\right)$ whenever $a<b$ .

Points in the domain of a function where the function changes from increasing to decreasing or from decreasing to increasing are called **critical points.**

**Sample Problem 4**: **Determine the interval(s) on which the function is increasing and the interval(s) on which the function is decreasing.**

|  |  |  |
| --- | --- | --- |
| **a.**  | $$f\left(x\right)=x^{2}-3x+2 $$ |  |
| **b.**  | $f\left(x\right)=x^{3}-\frac{1}{2}x^{2}-10x+2 $ |  |