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Analyzing Graphs of Functions and Relations

Unit 1 Lesson 2

Students will be able to:

Analyze graphs of functions and relations

(x and y – intercepts, zeros, symmetry, even and odd functions)

Key Vocabulary:

Graph of a function,

An intercept,

A zero of a function,

Symmetry

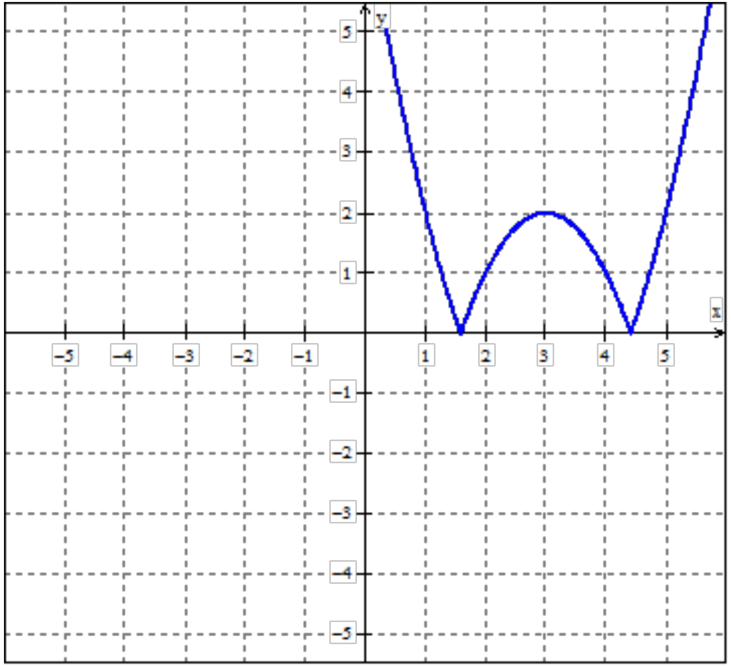
The graph of a function $\boldsymbol{f}$ is the set of ordered pairs$(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$**,** in the coordinate plane, such that $\boldsymbol{x}$ is the domain of $\boldsymbol{f}$.

- $\boldsymbol{x}$ – the directed distance from the $\boldsymbol{y}$ -axis
- $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ – the directed distance from the $\boldsymbol{x}$ -axis

You can use the graph to estimate function values.

Sample Problem 1: **Use a graph of each function to estimate the indicated function values. Then find the values algebraically.**

a. $f(x) = \left|(x-3)^2 - 2\right|$ $\quad f(3) = ?$ $\quad f(4) = ?$



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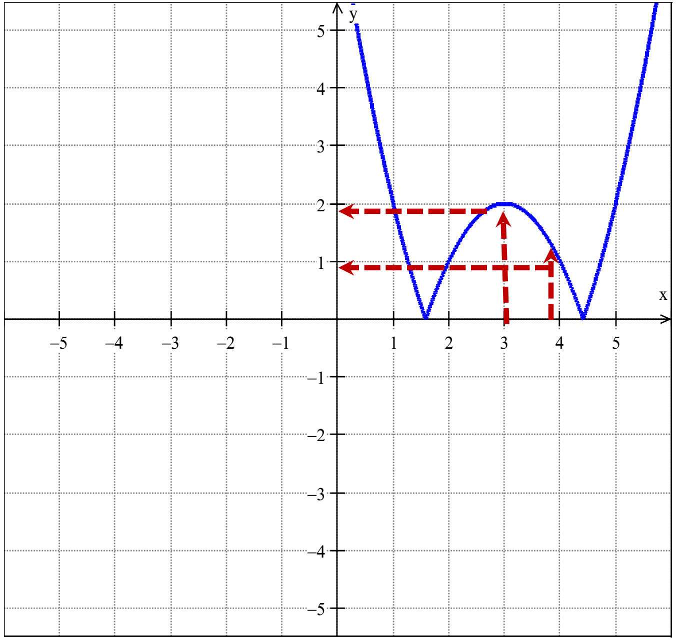


Graphically

$f(3) = 2$

$f(4) = 1$

Sample Problem 1: Use a graph of each function to estimate the indicated function values. Then find the values algebraically.

a. $f(x) = \left|(x-3)^2 - 2\right|$

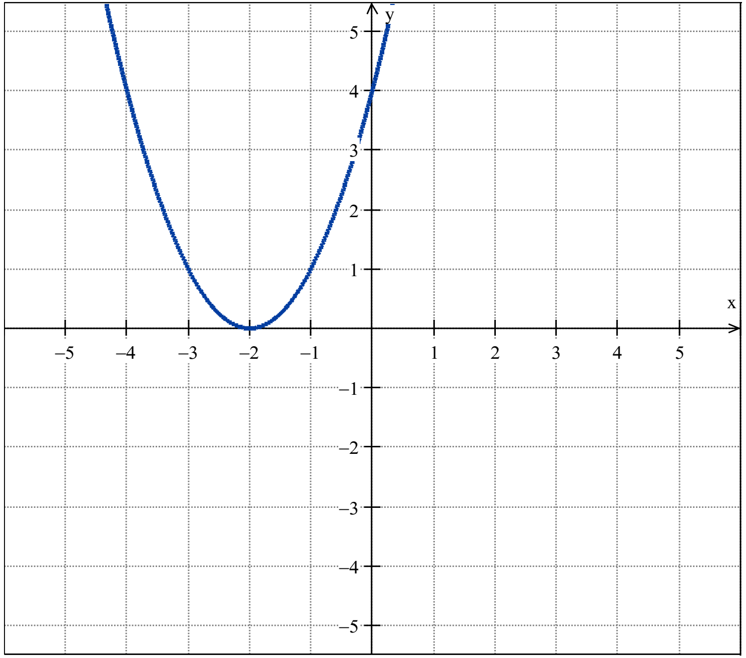
$Algebraically$

$f(3) = \left|(3-3)^2 - 2\right| = |0-2| = \mathbf{2}$

$f(4) = \left|(4-3)^2 - 2\right| = |1-2| = |-1| = \mathbf{1}$

Sample Problem 1: **Use a graph of each function to estimate the indicated function values. Then find the values algebraically.**

b. $f(x) = x^2 + 4x + 4$ $\quad f(3) = ?$ $\quad f(4) = ?$



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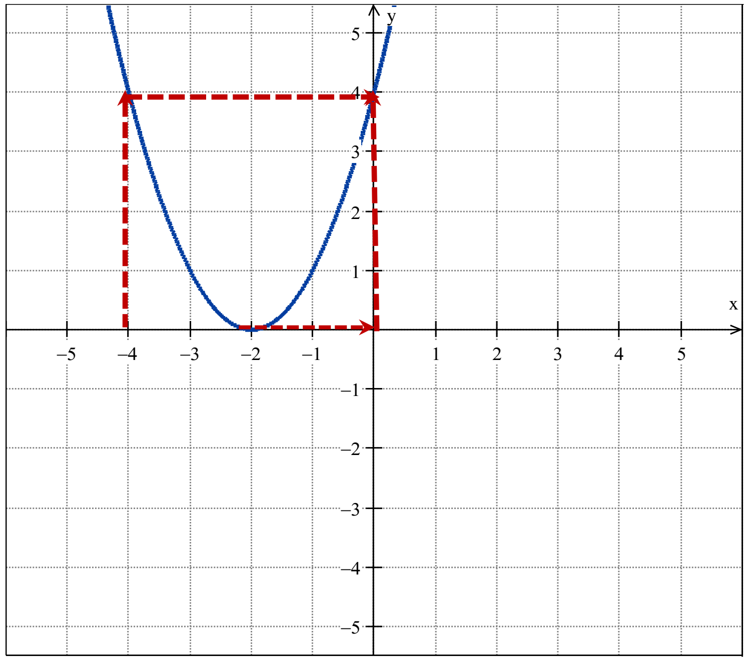


Graphically

$f(0) = 4$

$f(-2) = 0$

$f(-4) = 4$

Sample Problem 1: **Use a graph of each function to estimate the indicated function values. Then find the values algebraically.**

b. $f(x) = x^2 + 4x + 4$

Algebraically

$$f(0) = 0^2 + 4 * 0 + 4 = 4$$

$$f(-2) = (-2)^2 + 4(-2) + 4 = 4 - 8 + 4 = 0$$

$$f(-4) = (-4)^2 + 4(-4) + 4 = 16 - 16 + 4 = 4$$

Identifying Intercepts from a Functions Graph

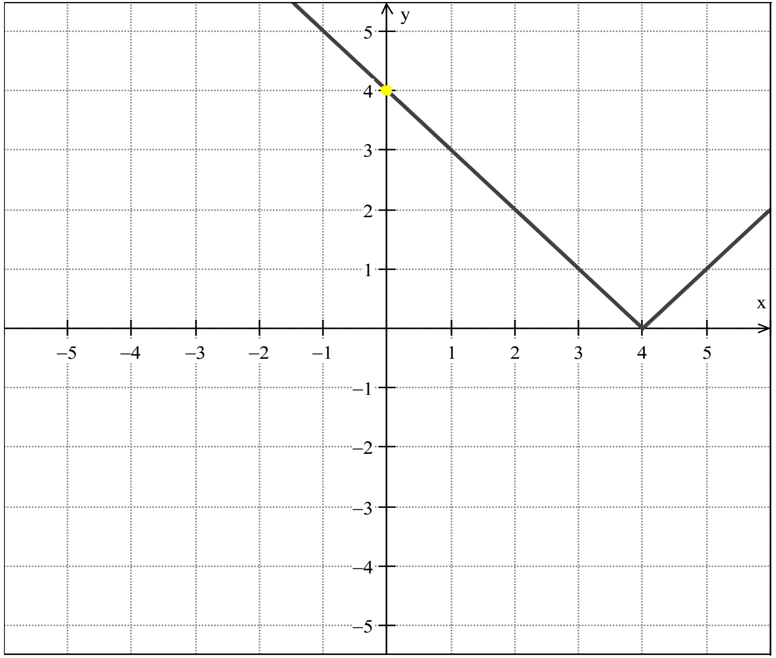
A point where the graph intersects or meets the $\boldsymbol{x}$ or $\boldsymbol{y}$ axis is called **an intercept.**

An $\boldsymbol{x}$ -intercept occurs where $\boldsymbol{y = 0}$.

A $\boldsymbol{y}$ -intercept occurs where $\boldsymbol{x = 0}$.

Sample Problem 2: Use the graph of each function to approximate its y –intercept. Then find the y –intercept algebraically.

a. $g(x) = |x - 4|$



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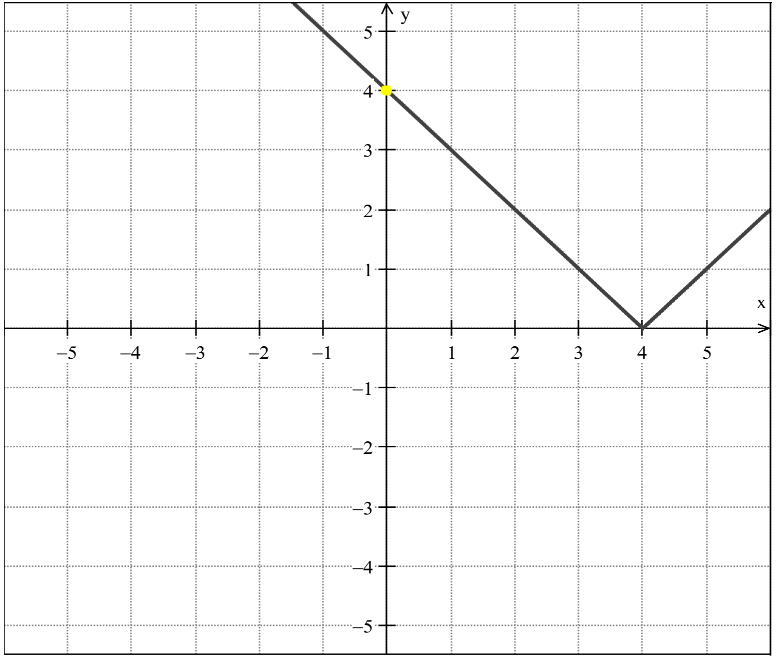


Graphically

$g(x) = |x - 4|$

y – intercept $= 4$

Sample Problem 2: Use the graph of each function to approximate its y –intercept. Then find the y –intercept algebraically.

a. $g(x) = |x - 4|$

Algebraically

y -intercept occurs where $x = 0$

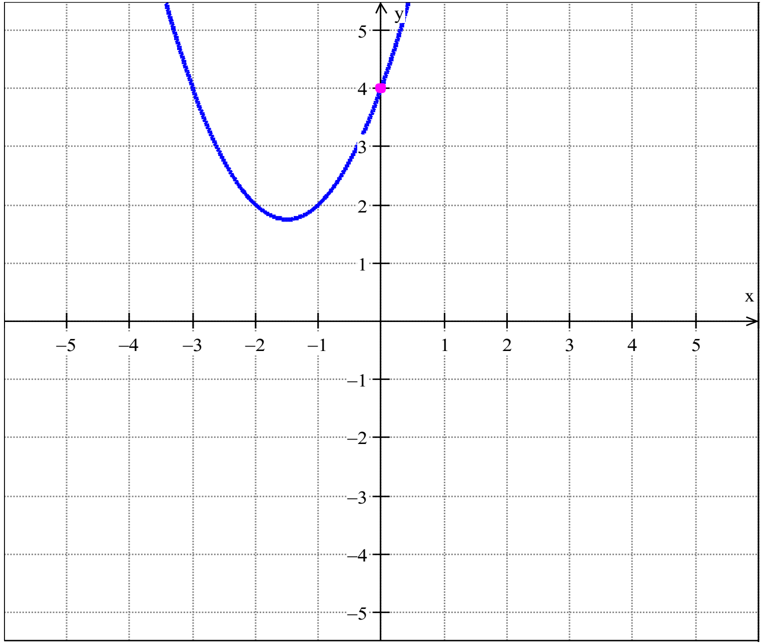
$g(0) = |0 - 4| = |-4|$

$g(0) = 4$

$y - \text{intercept} = 4$

Sample Problem 2: Use the graph of each function to approximate its y –intercept. Then find the y –intercept algebraically.

b. $f(x) = x^2 + 3x + 4$



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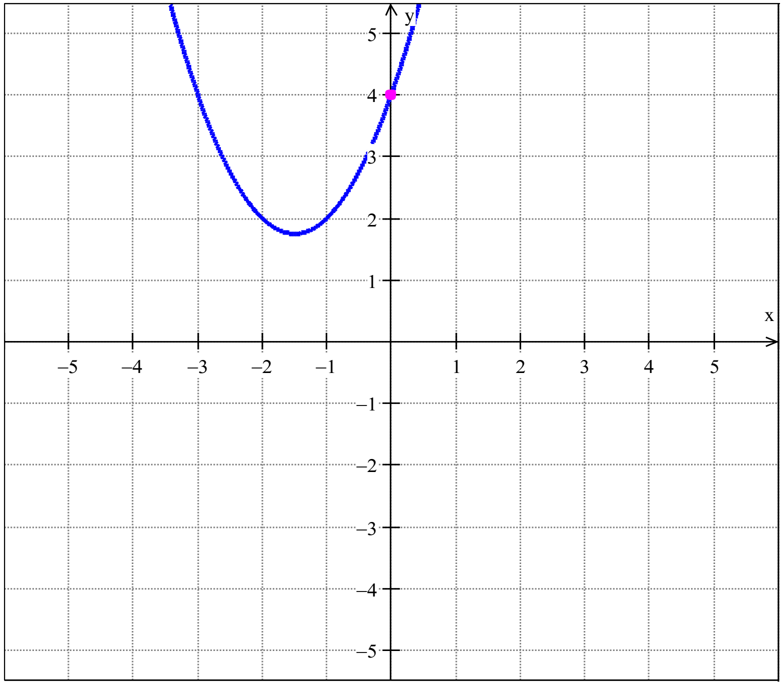


Graphically

$f(x) = x^2 + 3x + 4$

y – intercept $= 4$

Sample Problem 2: Use the graph of each function to approximate its y –intercept. Then find the y –intercept algebraically.

b. $f(x) = x^2 + 3x + 4$

$$Algebraically$$

y -intercept occurs where $x = 0$

$$f(0) = 0^2 + 3 * 0 + 4$$

$$f(0) = 4$$

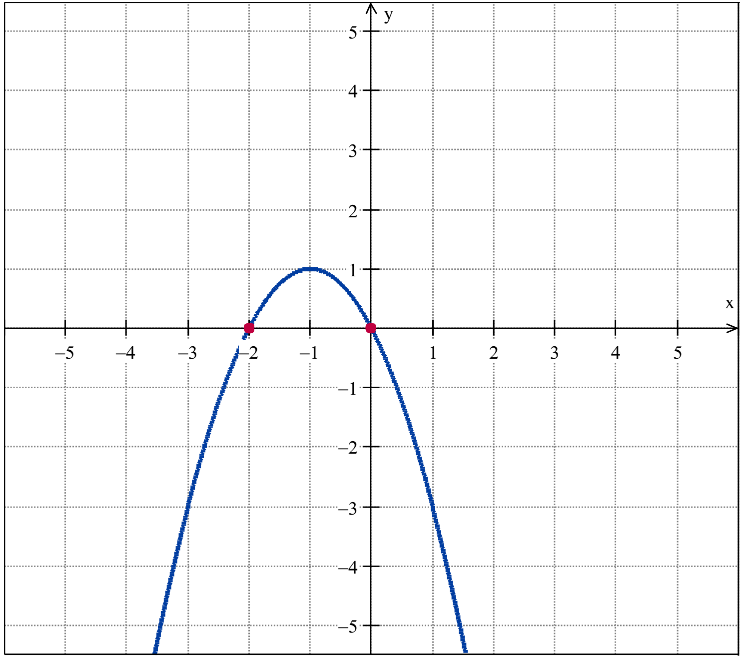
$$y - \text{intercept} = 4$$

Zeros of a Function

- The zeros of function $\boldsymbol{f(x)}$ are $\boldsymbol{x}$ –values for which $\boldsymbol{f(x) = 0}$
- If the graph of a function of $\boldsymbol{x}$ has an $\boldsymbol{x}$ -intercept at $(\boldsymbol{x}, \mathbf{0})$ then $\boldsymbol{x}$ is a zero of the function.
- To find the zeros of a function, set the function equal to zero and solve for the independent variable.

Sample Problem 3: **Use the graph of each function to approximate its zeros. Then find the zeros of each function algebraically.**

a. $f(x) = -x^2 - 2x$ $\quad$ $Zeros\ = ?$



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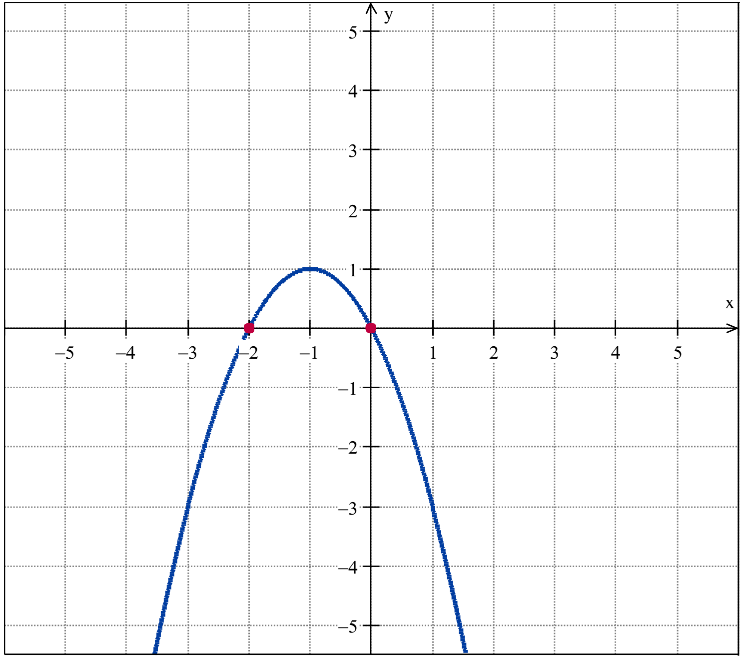


$Graphically$

$f(x) = -x^2 - 2x$

x – intercepts $\quad -2$ and 0

Sample Problem 3: **Use the graph of each function to approximate its zeros. Then find the zeros of each function algebraically.**

a. $f(x) = -x^2 - 2x \qquad Zeros\ =?$

$Algebraically$

$f(x) = 0$

$-x^2 - 2x = 0$

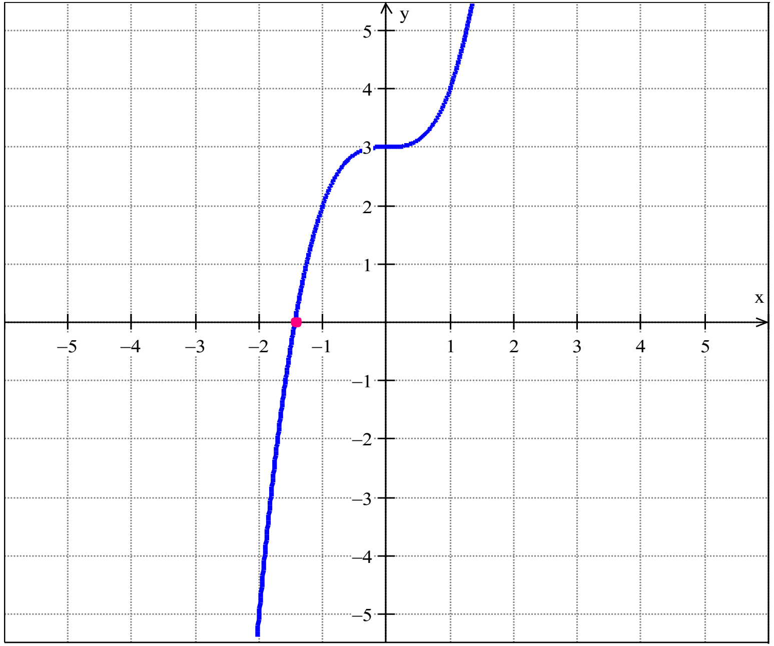
$-x(x+2) = 0$

$x = 0 \qquad or \qquad x = -2$

$The\ zeros\ of\ f\ \ are\ \ 0\ and - 2$

Sample Problem 3: **Use the graph of each function to approximate its zeros. Then find the zeros of each function algebraically.**

b. $f(x) = x^3 + 3$ $\quad\quad Zeros \; = ?$



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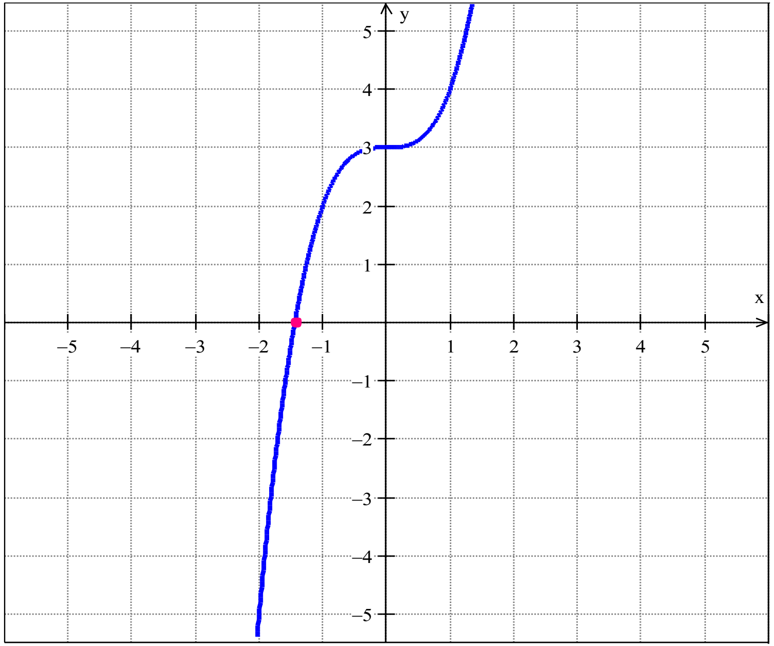


Graphically

$f(x) = x^3 + 3$

x – intercepts ≈ 1.3

Sample Problem 3: **Use the graph of each function to approximate its zeros. Then find the zeros of each function algebraically.**

b. $f(x) = x^3 + 3$ $\quad\quad Zeros\ = ?$

Algebraically

$f(x) = 0$

$x^3 + 3 = 0$

$x^3 = -3$

$x = \sqrt[3]{-3}$

$The\ zero\ of\ f\ is\ \sqrt[3]{-3} \approx -1.44$

Symmetry of Graphs

There are two possible types of symmetry that graphs of functions can have.

1. **Line symmetry** - graphs can be folded along a line so that the two halves match exactly.

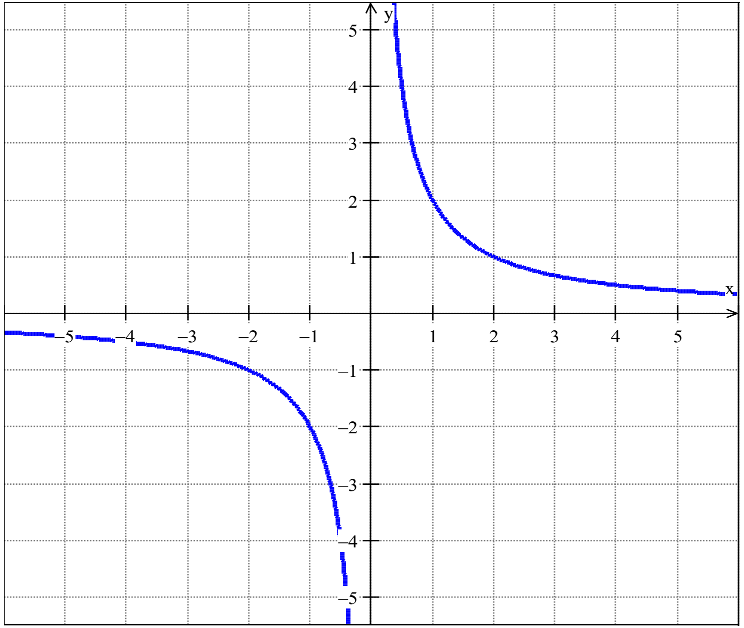
2. **Point symmetry** - graphs can be rotated 180° with respect to a point and appear unchanged.

Tests for Symmetry

Graphical Test	**Algebraic Test**
The graph of a relation is symmetric with respect to the x -axis if and only if for every point (x, y), on the graph, the point $(x, -y)$, is also on the graph.	Replacing y with - y produces an equivalent equation.
The graph of a relation is symmetric with respect to the y -axis if and only if for every point (x, y)on the graph, the point $(-x, y)$is also on the graph.	Replacing x with - x produces an equivalent equation.
The graph of a relation is symmetric with respect to the origin if and only if for every point (x, y) on the graph, the point $(-x, -y)$is also on the graph.	Replacing x with - x and y with - y produces an equivalent equation.

Sample Problem 4: **Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.**

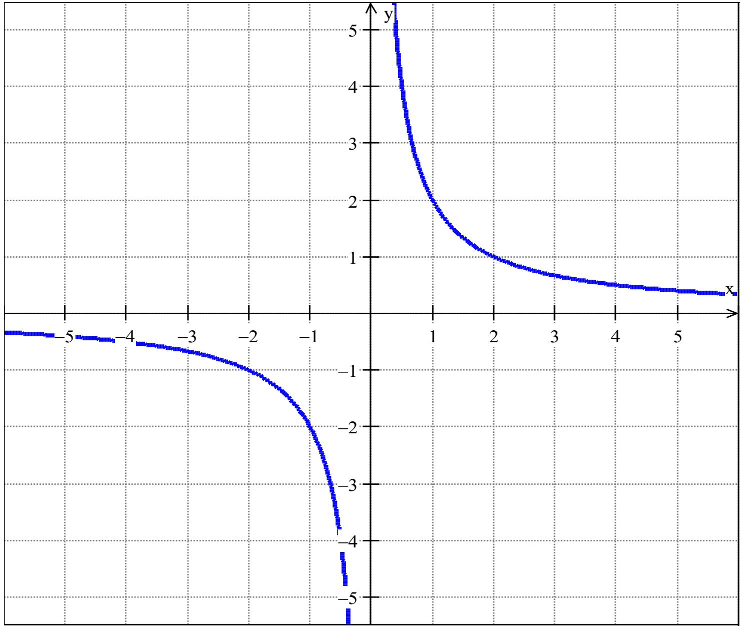
a. $y = \frac{2}{x}$



Sample Problem 4: **Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.**

a. $y = \frac{2}{x}$

<u>Graphically</u>



The graph appears to be symmetric with respect to the origin because for every point (x, y) on the graph, there is a point (- x,- y).

Sample Problem 4: Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.

a. $y = \frac{2}{x}$

Support Numerically

There is a table of values to support this conjecture.

x	-4	-2	-1	1	2	4
y	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$
(x, y)	$(-4, -\frac{1}{2})$	$(-2, -1)$	$(-1, -2)$	$(1, 2)$	$(2, 1)$	$(4, \frac{1}{2})$

Sample Problem 4: Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.

a. $y = \frac{2}{x}$

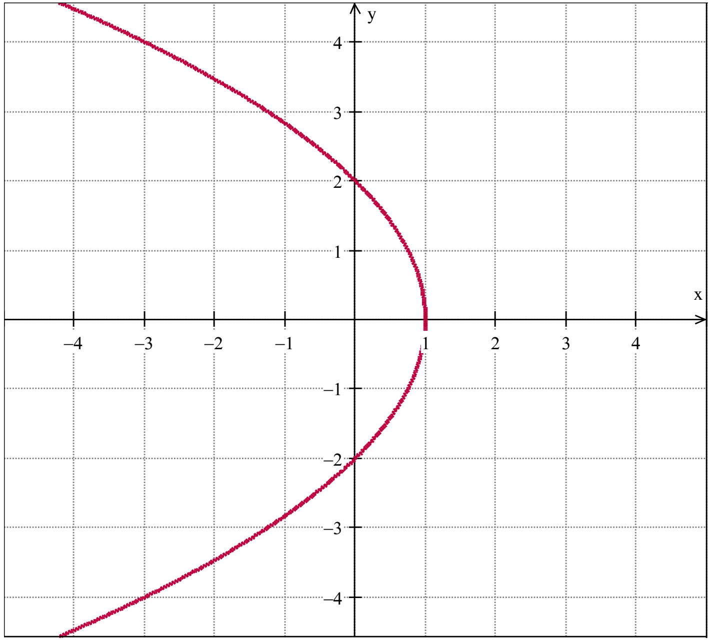
Algebraically

Because $-y = \frac{2}{-x}$ is equivalent to $y = \frac{2}{x}$,

the graph is symmetric with respect to the origin.

Sample Problem 4: **Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.**

b. $4x + y^2 = 4$

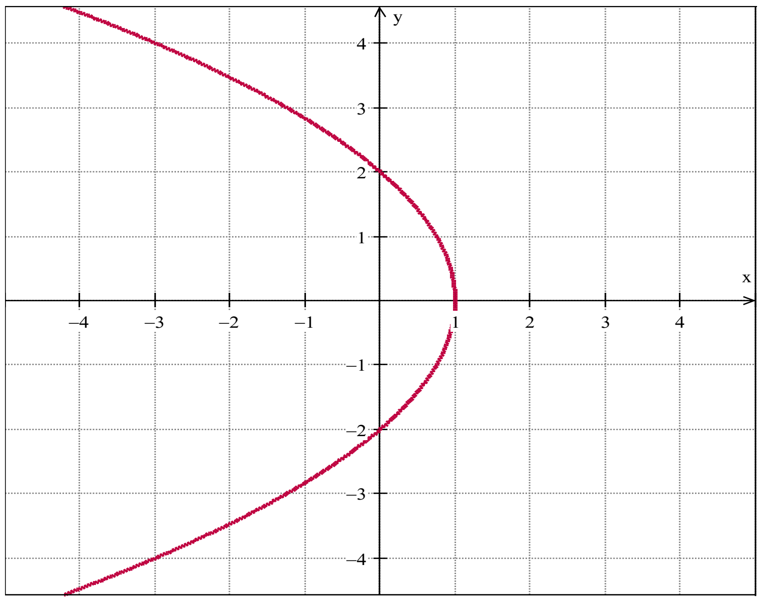


Sample Problem 4: Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.

b. $4x + y^2 = 4$

<u>**Graphically**</u>

The graph appears to be symmetric with respect to the x -axis because for every point (x, y) on the graph, there is a point $(x, -y)$.



Sample Problem 4: Use the graph of each equation to test for symmetry with respect to the x-axis, y-axis, and the origin. Support the answer numerically. Then confirm algebraically.

b. $4x + y^2 = 4$

<u>Support Numerically</u>

There is a table of values to support this conjecture.

x	-2	-1	0	1
y	$\pm 2\sqrt{3}$	$\pm 2\sqrt{2}$	± 2	0
(x, y)	$(-2, \pm 2\sqrt{3})$	$(-2, \pm 2\sqrt{2})$	$(-1, \pm 2)$	$(1, 0)$

Sample Problem 4: **Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.**

b. $4x + y^2 = 4$

Algebraically

$$4x + (-y)^2 = 4$$

$$4x + y^2 = 4$$

Because $4x + (-y)^2 = 4$ is equivalent to $4x + y^2 = 4$, the graph is symmetric with respect to the $x -$ axis.

Identify Even and Odd Functions

If $f(-x) = f(x)$, then the function is even, and symmetric to the y-axis.

If $f(-x) = -f(x)$, then the function is odd, and symmetric to the origin.

Sample Problem 5: Determine whether the following are even, odd, or neither.

a. $f(x) = x^4 + 4$

Sample Problem 5: Determine whether the following are even, odd, or neither.

a. $f(x) = x^4 + 4$

$f(-x) = (-x)^4 + 4$

$f(-x) = x^4 + 4$

$f(-x) = f(x)$ The function is even.

Sample Problem 5: Determine whether the following are even, odd, or neither.

b. $g(x) = 9x^5 - x^3$

Sample Problem 5: Determine whether the following are even, odd, or neither.

b. $g(x) = 9x^5 - x^3$

$g(-x) = 9(-x)^5 - (-x)^3$

$g(-x) = -9x^5 + x^3$

$g(-x) = -(9x^5 - x^3)$

$g(-x) = -g(x)$ The function is odd.

Sample Problem 5: **Determine whether the following are even, odd, or neither.**

c. $h(t) = t^2 + t$

Sample Problem 5: **Determine whether the following are even, odd, or neither.**

c. $h(t) = t^2 + t$

$$h(-t) = (-t)^2 + (-t)$$

$$h(-t) = t^2 - t$$

$$h(-t) \neq h(t) \qquad h(-t) \neq -h(t)$$

The function is neither.