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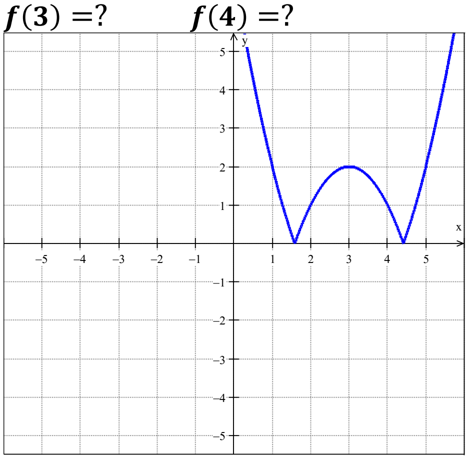
Analyzing Graphs of Functions and Relations Guided Notes

The graph of a function $\boldsymbol{f}$ is the set of ordered pairs$(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$, in the coordinate plane, such that $\boldsymbol{x}$ is the domain of $\boldsymbol{f}$.
$\boldsymbol{x} -$ the directed distance from the $\boldsymbol{y}$ -axis $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) -$ the directed distance from the $\boldsymbol{x}$ -axis

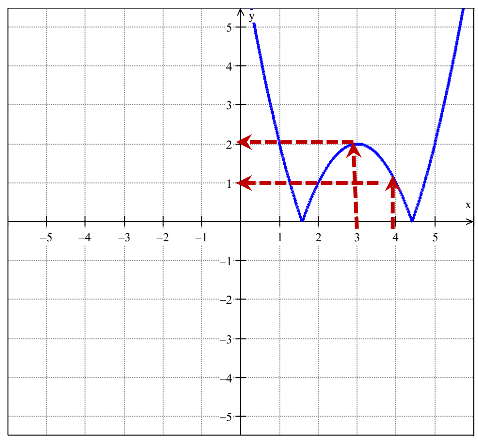
You can use the graph to estimate function values.

Sample Problem 1: Use a graph of each function to estimate the indicated function values. Then find the values algebraically.

a. $f(x) = |(x-3)^2 - 2|$
$f(3) = ?$ $f(4) = ?$

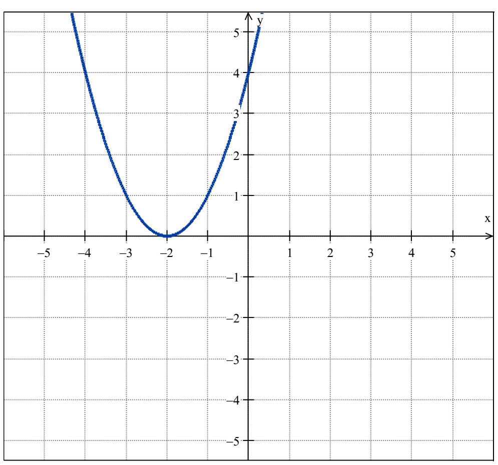


$\boldsymbol{Graphically}$ $f(3) = 2$ $f(4) = 1$

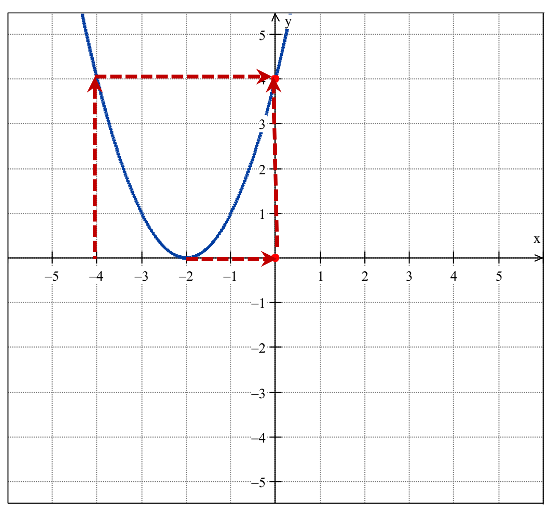


$\boldsymbol{Algebraically}$
$f(3) = |(3-3)^2 - 2| = |0-2| = 2$
$f(4) = |(4-3)^2 - 2| = |1-2| = |-1| = 1$

b. $f(x) = x^2 + 4x + 4$
$f(0) = ?$ $f(-2) = ?$ $f(-4) = ?$



$\boldsymbol{Graphically}$
$f(0) = 4$ $f(-2) = 0$ $f(-4) = 4$



$\boldsymbol{Algebraically}$
$f(0) = 0^2 + 4 * 0 + 4 = 4$
$f(-2) = (-2)^2 + 4(-2) + 4 = 4 - 8 + 4 = 0$
$f(-4) = (-4)^2 + 4(-4) + 4 = 16 - 16 + 4 = 4$

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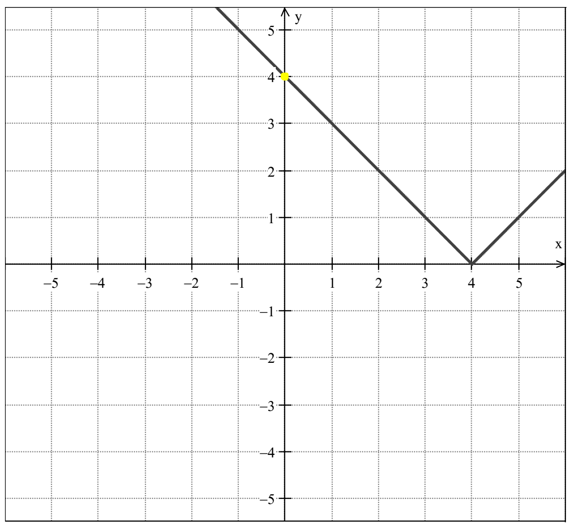
Identifying Intercepts from a Functions Graph

A point where the graph intersects or meets the x or y axis is called **an intercept.**

An x-intercept occurs where $y = 0$. A y-intercept occurs where $x = 0$.

Sample Problem 2: Use the graph of each function to approximate its y–intercept. Then find the y–intercept algebraically.

a. $g(x) = |x - 4|$



Graphically

$g(x) = |x - 4|$ $y - \text{intercept} = 4$

Algebraically

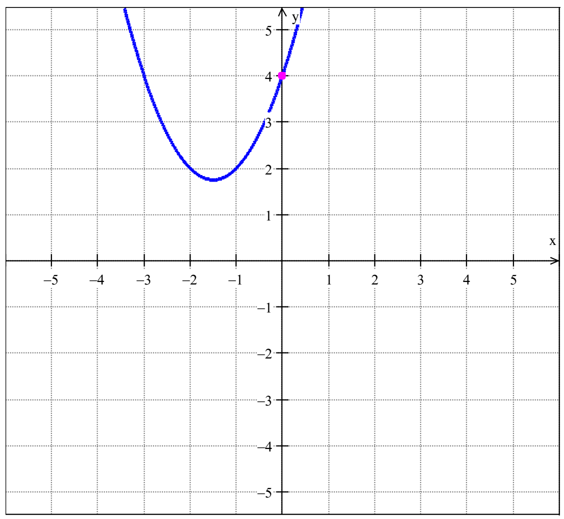
y-intercept occurs where $x = 0$.

$g(0) = |0 - 4| = |-4|$

$g(0) = 4$

$y - \text{intercept} = 4$

b. $f(x) = x^2 + 3x + 4$



Graphically

$f(x) = x^2 + 3x + 4$ $y - \text{intercept} = 4$

Algebraically

y-intercept occurs where $x = 0$.

$f(0) = 0^2 + 3 * 0 + 4$

$f(0) = 4$

$y - \text{intercept} = 4$

Zeros of a Function

The zeros of function $f(x)$ are x–values for which $f(x) = 0$

If the graph of a function of x has an x-intercept at $(x, 0)$ then x is a zero of the function.

To find the zeros of a function, set the function equal to zero and solve for the independent variable.

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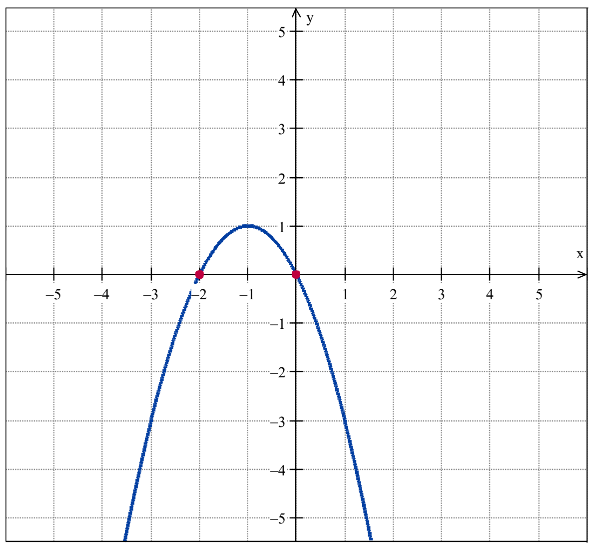
Sample Problem 3: Use the graph of each function to approximate its zeros. Then find the zeros of each function algebraically.

a. $f(x) = -x^2 - 2x$

$Zeros\ =?$

$f(x) = -x^2 - 2x$



$Graphically$

$f(x) = -x^2 - 2x$

x − intercepts -2 and 0

$Algebraically$

$f(x) = 0$

$-x^2 - 2x = 0$

$-x(x+2) = 0$

$x = 0$ $\quad or \quad$ $x + 2 = 0$

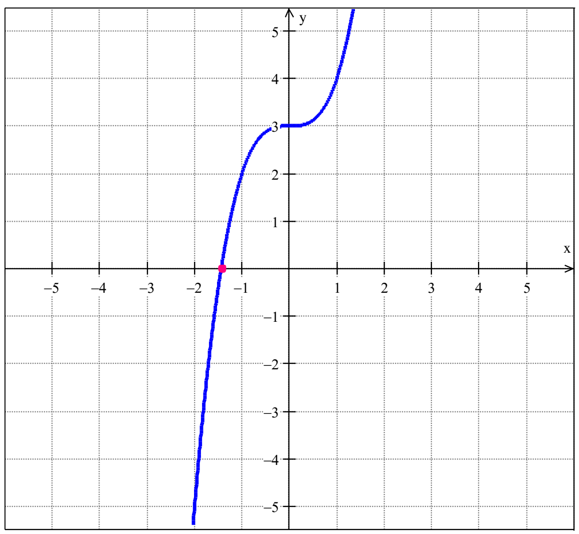
$x = -2$

$The\ zeros\ of\ f\ are$ $0\ and -2$

b. $f(x) = x^3 + 3$

$Zeros\ =?$

$f(x) = x^3 + 3$



$Graphically$

$f(x) = x^3 + 3$

x − intercepts $\approx$ 1.3

$Algebraically$

$f(x) = 0$

$x^3 + 3 = 0$

$x^3 = -3$

$x = \sqrt[3]{-3}$

$The\ zero\ of\ f\ is$ $\sqrt[3]{-3} \approx -1.44$

Symmetry of Graphs

There are two possible types of symmetry that graphs of functions can have.

1. **Line symmetry** - graphs can be folded along a line so that the two halves match exactly.

2. **Point symmetry** - graphs can be rotated 180° with respect to a point and appear unchanged.

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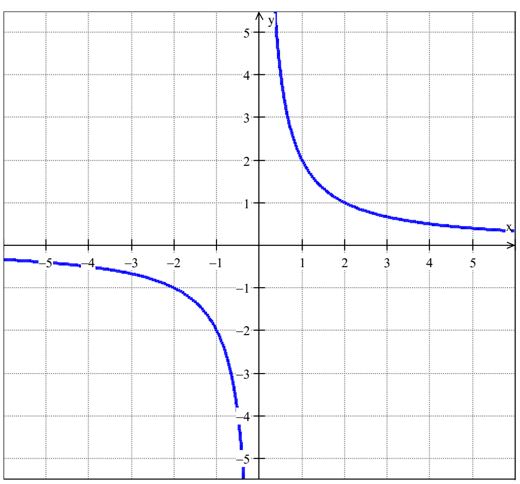
Tests for Symmetry

Graphical Test	**Algebraic Test**
The graph of a relation is symmetric with respect to the x -axis if and only if for every point (x, y), on the graph, the point $(x, -y)$, is also on the graph.	Replacing y with - y produces an equivalent equation.
The graph of a relation is symmetric with respect to the y -axis if and only if for every point (x, y)on the graph, the point $(-x, y)$is also on the graph.	Replacing x with - x produces an equivalent equation.
The graph of a relation is symmetric with respect to the origin if and only if for every point (x, y) on the graph, the point $(-x, -y)$is also on the graph.	Replacing x with - x and y with - y produces an equivalent equation.

Sample Problem 4: **Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.**

a. $y = \frac{2}{x}$



Graphically

The graph appears to be symmetric with respect to the origin because for every point (x, y) on the graph, there is a point $(-x, -y)$.

Support Numerically

There is a table of values to support this conjecture.

x	-4	-2	-1	1	2	4
y	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$
(x, y)	$(-4, -\frac{1}{2})$	$(-2, -1)$	$(-1, -2)$	$(1, 2)$	$(2, 1)$	$(4, \frac{1}{2})$

Algebraically

$$-y = \frac{2}{-x}$$

Because $-y = \frac{2}{-x}$ is equivalent to $y = \frac{2}{x}$, the graph is symmetric with respect to the origin.

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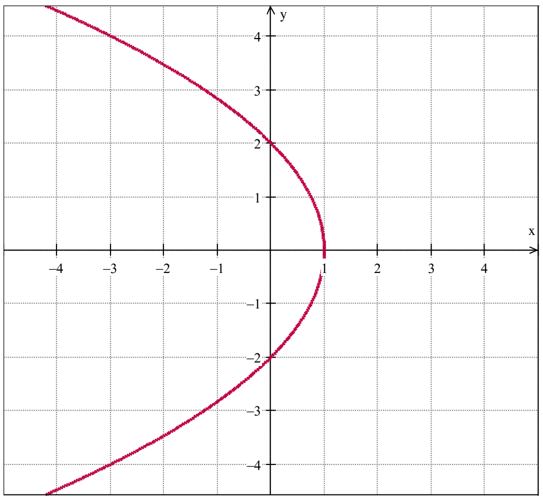
b. $4x + y^2 = 4$



Graphically

The graph appears to be symmetric with respect to the x -axis **because for every point (x, y) on the graph, there is a point $(x, -y)$.**

Support Numerically

There is a table of values to support this conjecture.

x	-2	-1	0	1
y	$\pm 2\sqrt{3}$	$\pm 2\sqrt{2}$	± 2	0
(x, y)	$(-2, \pm 2\sqrt{3})$	$(-2, \pm 2\sqrt{2})$	$(-1, \pm 2)$	$(1, 0)$

Algebraically

$4x + (-y)^2 = 4$
$4x + y^2 = 4$

Because
$4x + (-y)^2 = 4$ **is equivalent to** $4x + y^2 = 4$**, the graph is symmetric with respect** to the x -axis**.**

Identify Even and Odd Functions

If $f(-x) = f(x)$, then the function is even, and symmetric to the y-axis.

If $f(-x) = -f(x)$, then the function is odd, and symmetric to the origin.

Sample Problem 5: **Determine whether the following are even, odd, or neither.**

a. $f(x) = x^4 + 4$

$f(x) = x^4 + 4$
$f(-x) = (-x)^4 + 4$
$f(-x) = x^4 + 4$

$f(-x) = f(x)$ **The function is even.**

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b. $g(x) = 9x^5 - x^3$

$g(x) = 9x^5 - x^3$
$g(-x) = 9(-x)^5 - (-x)^3$
$g(-x) = -9x^5 + x^3$
$g(-x) = -(9x^5 - x^3)$

$g(-x) = -g(x)$ **The function is odd.**

c. $h(t) = t^2 + t$

$h(t) = t^2 + t$
$h(-t) = (-t)^2 + (-t)$
$h(-t) = t^2 - t$

$h(-t) \neq h(t)$ $h(-t) \neq -h(t)$
The function is neither.