The graph of a function $f$ is the set of ordered pairs$(x,f\left(x\right))$**,** in the coordinate plane, such that $x$ is the domain of $f$.

$x-$ the directed distance from the $y$ -axis $y=f\left(x\right)-$ the directed distance from the $x$ -axis

You can use the graph to estimate function values.

**Sample Problem 1: Use a graph of each function to estimate the indicated function values.** **Then find the values algebraically.**

|  |  |  |
| --- | --- | --- |
| **a.**  | $$f\left(x\right)=\left|\left(x-3\right)^{2}-2\right|$$$$f\left(3\right)=? f\left(4\right)=?$$ | $Graphically$$$Algebraically$$ |
| **b.**  | $$f\left(x\right)=x^{2}+4x+4$$$$f\left(0\right)=? f\left(-2\right)=? f\left(-4\right)=?$$ | $$Graphically$$$$Algebraically$$ |

**Identifying Intercepts from a Functions Graph**

A point where the graph intersects or meets the $x$ or $y$ axis is called **an intercept.**

An $x$ -intercept occurs where $y=0$. A $y$ -intercept occurs where $x=0$.

**Sample Problem 2**: Use the graph of each function to approximate its $y$ –intercept. Then find the $y$ –intercept algebraically.

|  |  |  |  |
| --- | --- | --- | --- |
| **a.**  | $$g\left(x\right)= \left|x-4\right|$$ | **b.**$$ $$ | $$f\left(x\right)= x^{2}+3x+4$$ |
|  | $$Graphically$$$$Algebraically $$ |  | $$Graphically$$$$Algebraically $$$$ $$ |

**Zeros of a Function**

The zeros of function $f\left(x\right) $are $x$ –values for which $f\left(x\right)=0$

If the graph of a function of $x$ has an $x$ -intercept at $(x,0)$ then $x$ is a zero of the function.

To find the zeros of a function, set the function equal to zero and solve for the independent variable.

**Sample Problem 3**: **Use the graph of each function to approximate its zeros. Then find the zeros of each function algebraically.**

|  |  |  |  |
| --- | --- | --- | --- |
| **a.** | $$f\left(x\right)=-x^{2}-2x$$$$Zeros =?$$ | **b.** | $$f\left(x\right)=x^{3}+3$$$$Zeros =?$$ |
|  | $$f\left(x\right)= -x^{2}-2x$$ |  | $$f\left(x\right)= x^{3}+3$$ |
|  | $$Graphically$$$$Algebraically $$ |  | $$Graphically$$$$Algebraically $$ |

**Symmetry of Graphs**

There are two possible types of symmetry that graphs of functions can have.

1. **Line symmetry** - graphs can be folded along a line so that the two halves match exactly.

2. **Point symmetry** - graphs can be rotated 180° with respect to a point and appear unchanged.

**Tests for Symmetry**

|  |  |
| --- | --- |
| **Graphical Test** | **Algebraic Test** |
| The graph of a relation is symmetric with respect to the $x$ -axis if and only if for every point ($x,y)$, on the graph, the point ($x,-y)$, is also on the graph. | Replacing $y$ with -$ y$ produces an equivalent equation. |
| The graph of a relation is symmetric with respect to the $ y$ -axis if and only if for every point ($x,y)$on the graph, the point ($-x,y)$is also on the graph. | Replacing $x$ with -$ x$ produces an equivalent equation. |
| The graph of a relation is symmetric with respect to the origin if and only if for every point ($x,y)$ on the graph, the point ($-x,-y)$is also on the graph. | Replacing $x$ with -$ x$ and $y$ with -$ y$ produces an equivalent equation. |

**Sample Problem 4**: **Use the graph of each equation to test for symmetry with respect to the** $x$ **-axis,** $y$ **-axis, and the origin.** **Support the answer numerically. Then confirm algebraically.**

|  |  |
| --- | --- |
| **a.** | $$y=\frac{2}{x}$$ |
|  |  | **Graphically****Support Numerically**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |  |  |
| $$y$$ |  |  |  |  |  |  |
| $$(x,y)$$ |  |  |  |  |  |  |

**Algebraically** |
| **b.** | $$4x+y^{2}=4$$ |
|  |  | **Graphically****Support Numerically**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$y$$ |  |  |  |  |
| $$(x,y)$$ |  |  |  |  |

**Algebraically** |

**Identify Even and Odd Functions**

If $f\left(-x\right)=f\left(x\right),$ then the function is even, and symmetric to the y-axis.

If $f\left(-x\right)=-f\left(x\right), $then the function is odd, and symmetric to the origin.

**Sample Problem 5**: **Determine whether the following are even, odd, or neither.**

|  |  |  |
| --- | --- | --- |
| **a.** | $$f\left(x\right)=x^{4}+4$$ |  |
| **b.** | $g\left(x\right)=9x^{5}-x^{3}$ |  |
| **c.** | $$h\left(t\right)=t^{2}+t$$ |  |