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Functions

Unit 1 Lesson 1

Students will be able to:

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Key Vocabulary:

Function, input, output, domain, range

A function *f* from set *A* to set *B* is a relation that assigns to each element *x* in set *A* exactly one element *y* in set *B*.

The notation of function is f(x).

The variable *x* - input value is called **the independent variable**, and *y* - output value is called **the dependent variable**.

- The set *A* of inputs is called **the domain** of the function *f*.
- The set *B* of all conceivable outputs is the codomain of the function *f*. The set of all outputs is the range of *f*. The range is a subset of *B*.
- The most important criteria for a function is this:
- "A function must assign to each input a unique output"

- If in a relation each input corresponds to exactly one output, and every output corresponds to exactly one input, this kind of relation is **a one-to-one function**.
- A function is usually specified numerically using a table of values, graphically using a graph, or algebraically using a formula. The graph of a function consists of all points (x, f(x)) in the plane.

Sample Problem 1: Determine whether each relation is a function.





Sample Problem 1: Determine whether each relation is a function.



 $A = \{2, 4, 3\}$ $B = \{3, -2, 7\}$ $R_1 = \{(2,3); (4,-2); (3,7)\}$ Each element of **A** HAS unique output in **B**. The relation is **A FUNCTION.** It is **A ONE TO ONE FUNCTION. f**PreCalculusCoach.com

Sample Problem 1: Determine whether each relation is a function.





Sample Problem 1: Determine whether each relation is a function.



 $S = \{1, 2, 6\}$ $T = \{5, 6, 9\}$ $R_2 = \{(1, 5); (1, 6); (2, 9); (6, 9)\}$ Each element of S has NOT unique output in T. The relation is **NOT A FUNCTION.**

The Vertical Line Test:

A set of points in the plane represents y as a function of x, if and only if no two points lie on the same vertical line.



Sample Problem 2: Use the Vertical Line Test to determine which of the following graphs describes *y* as a function of *x*.

a. 5 -5 -3 -2 X -4 4

-3

-4



Sample Problem 2: Use the Vertical Line Test to determine which of the following graphs describes y as a function of x.



In the graph, vertical line x = 3crosses the graph at most once, so graph does represent y as a x function of x.

Sample Problem 2: Use the Vertical Line Test to determine which of the following graphs describes y as a function of x.





Sample Problem 2: Use the Vertical Line Test to determine which of the following graphs describes y as a function of x.



In the graph, vertical line x = -2crosses the graph more than once, so graph does not represent y as a x function of x.

Evaluate and Solve the Function

- When we know an input value and want to determine the corresponding output value for a function, we evaluate the function.
- Evaluating will always produce one result because each input value of a function corresponds to exactly one output value.



 When we know an output value and want to determine the input values that would produce that output value, we set the output equal to the function's formula and solve for the input.



a.
$$f(x) = x^2 + 2x + 3$$

$$f(-2) = ?$$



a.
$$f(x) = x^2 + 2x + 3$$

 $f(-2) = (-2)^2 + 2 * (-2) + 3$
 $f(-2) = 4 + (-4) + 3$
 $f(-2) = 0 + 3$
 $f(-2) = 3$



b.
$$f(x) = 3x^2 + 3$$

$$f(a) = ?$$



b.
$$f(x) = 3x^2 + 3$$

 $f(a) = 3a^2 + 3$



c.
$$g(x) = \sqrt{x-3}$$

$$g(a+3) = ?$$



c.
$$g(x) = \sqrt{x-3}$$
$$g(a+3) = \sqrt{(a+3)-3}$$
$$g(a+3) = \sqrt{a}$$



Sample Problem 3: Evaluate each function.

d.
$$f(x) = 2x + 6$$

$$f(-1) = ?$$

f(x) = 12



Sample Problem 3: Evaluate each function.

d.
$$f(x) = 2x + 6$$

$$f(x) = 12$$

2x + 6 = 12
2x + 6 - 6 = 12 - 6
2x = 6
x = 3

The Domain and the Range of the Functions

There are several ways to write the domain and the range of the function.



The Domain and the Range of the Functions

1. Inequality Notation

For example: **0** < *x* < **10**

2. Set-builder Notation is a method of specifying a set of elements that satisfy a certain condition.

It takes the form: { *x*|*statement of x*}.

For example: $\{ x \mid x \leq 8, x \in R \}$



The Domain and Range of the Functions

- **3. Interval Notation** is a way of describing sets that include all real numbers between a lower limit that may or may not be included and an upper limit that may or may not be included. The endpoint values are listed between brackets or parentheses. A square bracket indicates inclusion in the set, and a parenthesis indicates exclusion from the set.
- For example: $[8, \infty)$



Domains can be restricted if:

- The function is a rational function and the denominator is 0 for some value or values of *x*.
- The function is a radical function with an even index (such as a square root), and the radicand can be negative for some value or values of *x*.



Sample Problem 4: Find the domain and range of each

function. Write in interval notation.



x		
У		



Sample Problem 4: Find the domain and range of each

function. Write in interval notation.



x	0	-1	1
у	3	1	5

Domain = $(-\infty, \infty)$

$$Range = (-\infty, \infty)$$

Sample Problem 4: Find the domain and range of each

function. Write in interval notation.



x		
У		



Sample Problem 4: Find the domain and range of each

function. Write in interval notation.



x	0	-1	1
у	2	5	5

Domain = $(-\infty, \infty)$ **Range** = $[2, \infty)$

Sample Problem 4: Find the domain and range of each function.

Write in interval notation.



x			
у			



Sample Problem 4: Find the domain and range of each function.

Write in interval notation.



x	-4	-3	-1	0	1	2
у	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

 $x + 2 \neq 0 \qquad x \neq -2$ Domain = $(-\infty, -2) \cup (-2, \infty)$ Range = $(-\infty, 0) \cup (0, \infty)$

Sample Problem 4: Find the domain and range of each function.

Write in interval notation.



x		
у		



Sample Problem 4: Find the domain and range of each function.

Write in interval notation.



x	-3	0	6
у	0	1.73	3

 $x + 3 \ge 0$ $x \ge -3$ $Domain = [-3, \infty)$ $Range = [0, \infty)$ $\int PreCalculusCoach.com$

Domain

Piecewise Functions

- A piecewise function is a function in which more than one formula is used to define the output.
- Each formula has its own domain, and the domain of the function is the union of all these smaller domains.

$$f(x) = \begin{cases} formula 1, & if \\ formula 2, & if \\ formula 3, & if \end{cases}$$

Domain to use formula 1 Domain to use formula 2 Domain to use formula 3

a.
$$f(x) = \begin{cases} 2x + 4, & \text{if } x < 0 \\ x^2 + x, & \text{if } x \ge 0 \end{cases}$$

 $f(4) = ?$



a.
$$f(x) = \begin{cases} 2x + 4, & \text{if } x < 0\\ x^2 + x, & \text{if } x \ge 0 \end{cases}$$

To find f(4) use $f(x) = x^2 + x$
 $f(4) = 4^2 + 4$
 $f(4) = 16 + 4$
 $f(4) = 20$



b.
$$f(x) = \begin{cases} -2x+4, & \text{if } x < -1 \\ -x^2, & \text{if } -1 < x < 5 \\ 2x^3, & \text{if } x > 5 \end{cases}$$
$$f(-2) = ?$$
$$f(3) = ?$$



Sample Problem 5: Evaluate each piecewise function. b. $f(x) = \begin{cases} -2x+4, & if \quad x < -1 \\ -x^2, & if \quad -1 < x < 5 \\ 2x^3, & if \quad x > 5 \end{cases}$ *To find* f(-2) *use* f(x) = -2x + 4f(-2) = -2 * (-2) + 4f(-2) = 4 + 4

f(-2) = 8



b.
$$f(x) = \begin{cases} -2x + 4, & if \quad x < -1 \\ -x^2, & if \quad -1 < x < 5 \\ 2x^3, & if \quad x > 5 \end{cases}$$

To find
$$f(3)$$
 use $f(x) = -x^2$
 $f(3) = -3^2$
 $f(3) = -9$



Sample Problem 6: Find the domain and range of each function.

Write in interval notation.





Sample Problem 6: Find the domain and range of each function.

Write in interval notation.



$$f(x) = -x, \quad x < 0$$

$$D_1 = (-\infty, 0)$$

$$f(x) = x, \quad x \ge 0$$

$$D_2 = [0, \infty)$$

$$D = D_1 \cup D_2 = (-\infty, \infty)$$

$$R = [0, \infty)$$

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Domain

Sample Problem 6: Find the domain and range of each function.

Write in interval notation. **b.** $f(x) = \begin{cases} x+1, & x>0\\ x, & x \leq 0 \end{cases}$ 5 4 -4 -3 -5 X -3 -4 -5



Sample Problem 6: Find the domain and range of each function.

Write in interval notation. $x \leq 0$ **b.** f(x) =Х, x + 1x > 05 4 R a -4 -3 -5 n X g е -3 -4 -5

$$f(x) = x, \qquad x \le 0$$

$$D_1 = (-\infty, 0]$$

$$f(x) = x + 1, \qquad x > 0$$

$$D_2 = (0, \infty)$$

$$D = D_1 \cup D_2 = (-\infty, \infty)$$

$$R = (-\infty, 0] \cup (1, \infty)$$
Domain
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