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Functions Guided Notes

A function f from set A to set B is a relation that assigns to each element x in set A exactly one element y in set B.

The notation of function is $f(x)$.

The variable x - input value is called **the independent variable**, and y - output value is called **the dependent variable**.

The set A of inputs is called **the domain** of the function f.

The set B of all conceivable outputs is **the codomain** of the function f. The set of all outputs is **the range of f**. The range is a subset of B.

The most important criteria for a function is this:

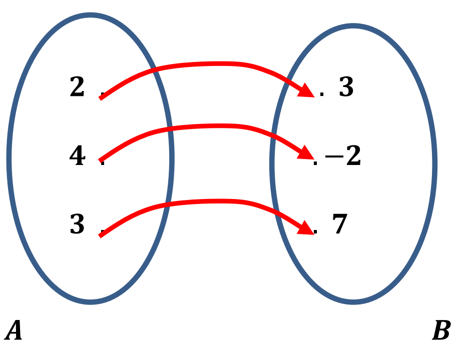
"A function must assign to each input a unique output"

If in a relation each input corresponds to exactly one output, and every output corresponds to exactly one input, this kind of relation is **a one-to-one function.**

A function is usually specified **numerically** using a table of values, **graphically** using a graph, or **algebraically** using a formula. The graph of a function consists of all points $(x, f(x))$ in the plane.

Sample Problem 1: Determine whether each relation is a function.

a.

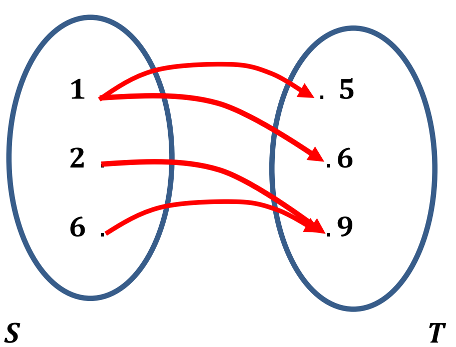


$A = \{2, 4, 3\}$
$B = \{3, -2, 7\}$

$R_1 = \{(2, 3); (4, -2); (3, 7)\}$

Each element of A **HAS** unique output in B
The relation is **A FUNCTION.**
It is **A ONE TO ONE FUNCTION.**

b.



$S = \{1, 2, 6\}$
$T = \{5, 6, 9\}$

$R_2 = \{(1, 5); (1, 6); (2, 9); (6, 9)\}$

Each element of S has **NOT** unique output in T
The relation is **NOT A FUNCTION.**

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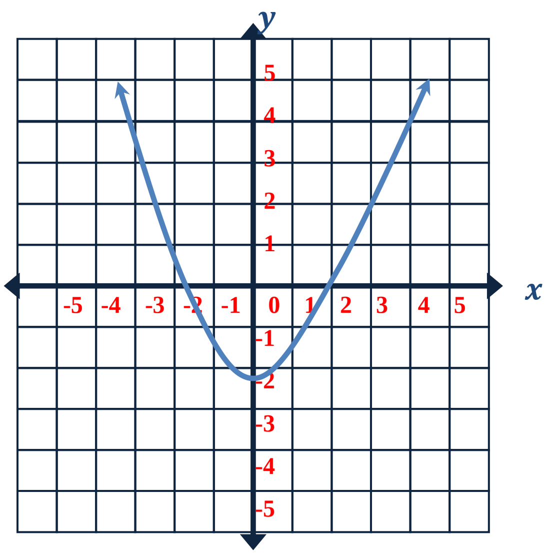
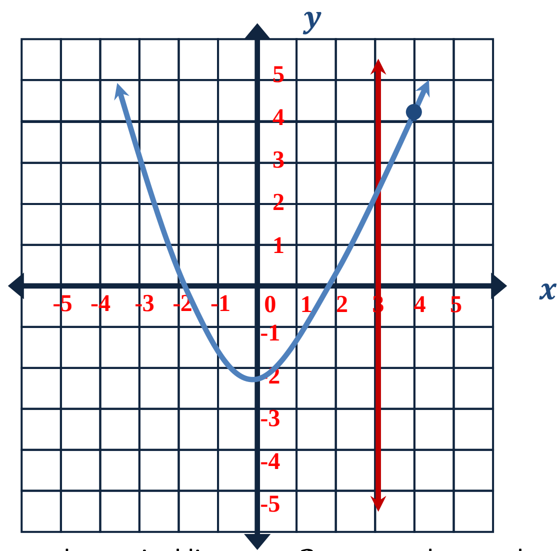
Functions Guided Notes

The Vertical Line Test: A set of points in the plane represents y as a function of x, if and only if no two points lie on the same vertical line.

Sample Problem 2: Use the Vertical Line Test to determine which of the following graphs describes y as a function of x.

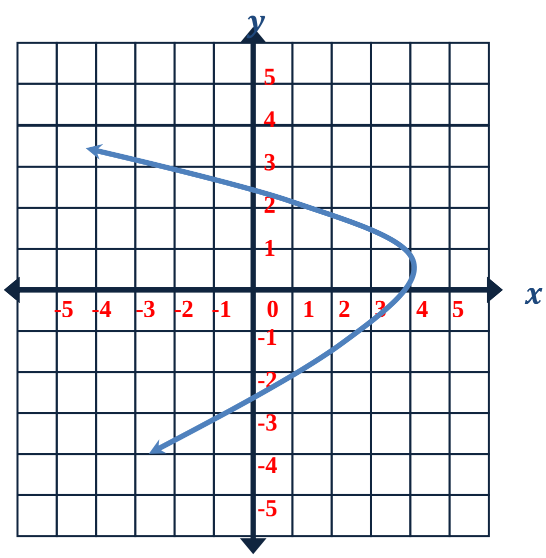
a.

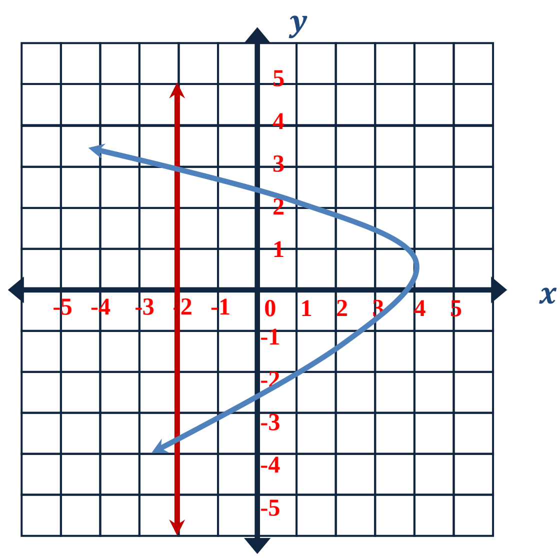




In the graph, vertical line $x = 3$ crosses the graph at most once, so graph does represent y as a function of x.

b.





In the graph, vertical line $x = -2$ crosses the graph more than once, so graph does not represent y as a function of x.

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Functions Guided Notes

Evaluate and Solve the Function

When we know an input value and want to determine the corresponding output value for a function, we evaluate the function. Evaluating will always produce one result because each input value of a function corresponds to exactly one output value.

When we know an output value and want to determine the input values that would produce that output value, we set the output equal to the function's formula and solve for the input.

Sample Problem 3: Evaluate each function.

a.

$f(x) = x^2 + 2x + 3$
$f(-2) = ?$

$f(-2) = (-2)^2 + 2 * (-2) + 3$
$f(-2) = 4 + (-4) + 3$
$f(-2) = 0 + 3$
$f(-2) = 3$

b.

$f(x) = 3x^2 + 3$
$f(a) = ?$

$f(a) = 3a^2 + 3$

c.

$g(x) = \sqrt{x - 3}$
$g(a + 3) = ?$

$g(a + 3) = \sqrt{(a + 3) - 3}$
$g(a + 3) = \sqrt{a}$

d.

$f(x) = 2x + 6$
$f(-1) = ?$
$f(x) = 12$

$f(-1) = 2 * (-1) + 6$
$f(-1) = -2 + 6$
$f(-1) = 4$

$f(x) = 12$
$2x + 6 = 12$
$2x + 6 - 6 = 12 - 6$
$2x = 6$
$x = 3$

The Domain and the Range of the Functions

There are several ways to write the domain and the range of the function.

Inequality Notation - For example: $0 < x < 10$

Set-builder Notation is a method of specifying a set of elements that satisfy a certain condition. It takes the form: $\{x | statement\ of\ x\}$. For example: $\{x | x \leq 8,\ x \in R\}$

Interval Notation is a way of describing sets that include all real numbers between a lower limit that may or may not be included and an upper limit that may or may not be included. The endpoint values are listed between brackets or parentheses. A square bracket indicates inclusion in the set, and a parenthesis indicates exclusion from the set.

For example: $[8, \infty)$

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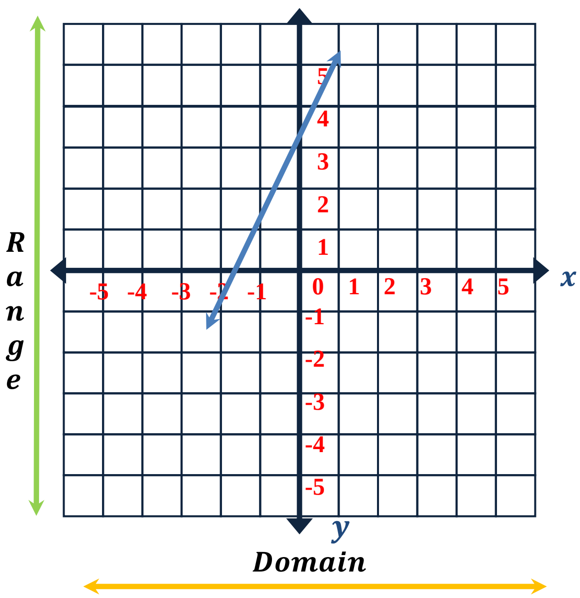
Domains can be restricted if:

- The function is a rational function and the denominator is 0 for some value or values of x.
- The function is a radical function with an even index (such as a square root), and the radicand can be negative for some value or values of x.

Sample Problem 4: Find the domain and range of each function. Write in interval notation.

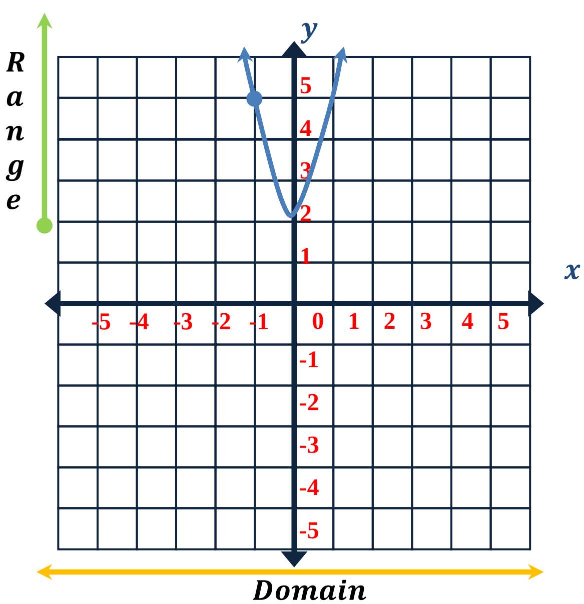
a. $f(x) = 2x + 3$



x	0	-1	1
y	3	1	5

$Domain = (-\infty, \infty)$
$Range = (-\infty, \infty)$

b. $f(x) = 3x^2 + 2$



x	0	-1	1
y	2	5	5

$Domain = (-\infty, \infty)$
$Range = [2, \infty)$

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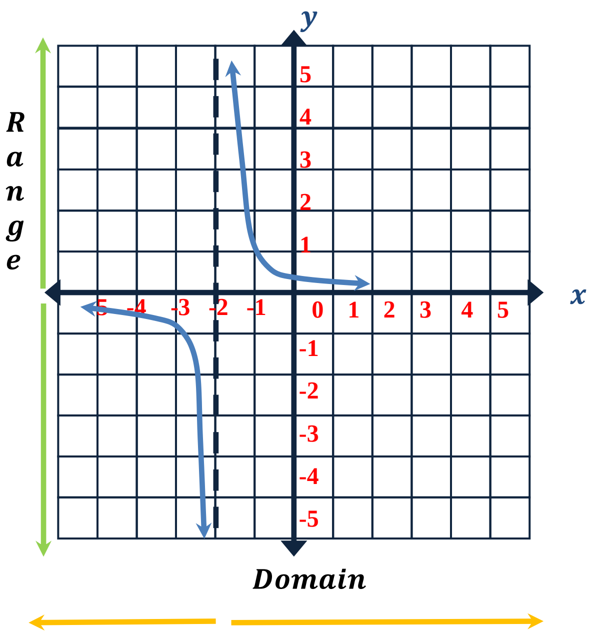
c. $f(x) = \dfrac{1}{x+2}$



x	-4	-3	-1	0	1	2
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

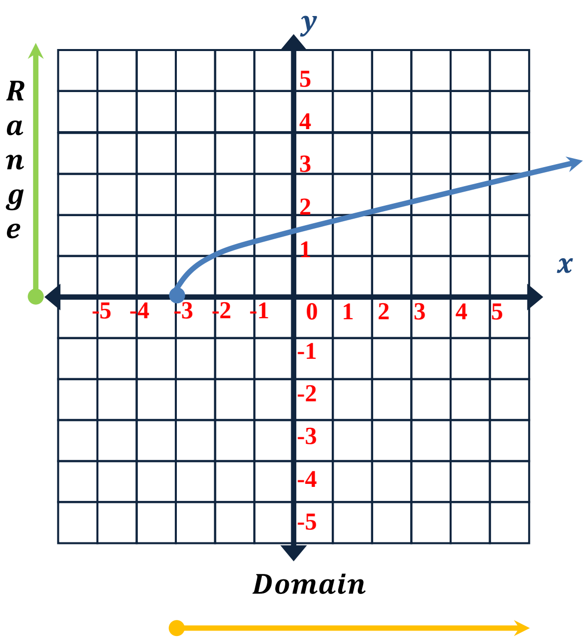
$x + 2 \neq 0$

$x \neq -2$

$Domain = (-\infty, -2) \cup (-2, \infty)$

$Range = (-\infty, 0) \cup (0, \infty)$

d. $f(x) = \sqrt{x+3}$



x	-3	0	6
y	0	1.73	3

$x + 3 \geq 0$

$x \geq -3$

$Domain = [-3, \infty)$

$Range = [0, \infty)$

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Functions Guided Notes

Piecewise Functions

A piecewise function is a function in which more than one formula is used to define the output. Each formula has its own domain, and the domain of the function is the union of all these smaller domains.

$$f(x) = \begin{cases} formula\ 1, & if \quad Domain\ to\ use\ formula\ 1 \\ formula\ 2, & if \quad Domain\ to\ use\ formula\ 2 \\ formula\ 3, & if \quad Domain\ to\ use\ formula\ 3 \end{cases}$$

Sample Problem 5: Evaluate each piecewise function.

a. $f(x) = \begin{cases} 2x+4, & if\ x < 0 \\ x^2 + x, & if\ x \geq 0 \end{cases}$

$f(4) = ?$

$To\ find\ f(4)\ use\ f(x) = x^2 + x$

$f(4) = 4^2 + 4$

$f(4) = 16 + 4$

$f(4) = 20$

b. $f(x) = \begin{cases} -2x+4, & if \quad x < -1 \\ -x^2, & if \quad -1 < x < 5 \\ 2x^3, & if \quad x > 5 \end{cases}$

$f(-2) = ?$

$f(3) = ?$

$To\ find\ f(-2)\ use\ f(x) = -2x + 4$

$f(-2) = -2 * (-2) + 4$

$f(-2) = 4 + 4$

$f(-2) = 8$

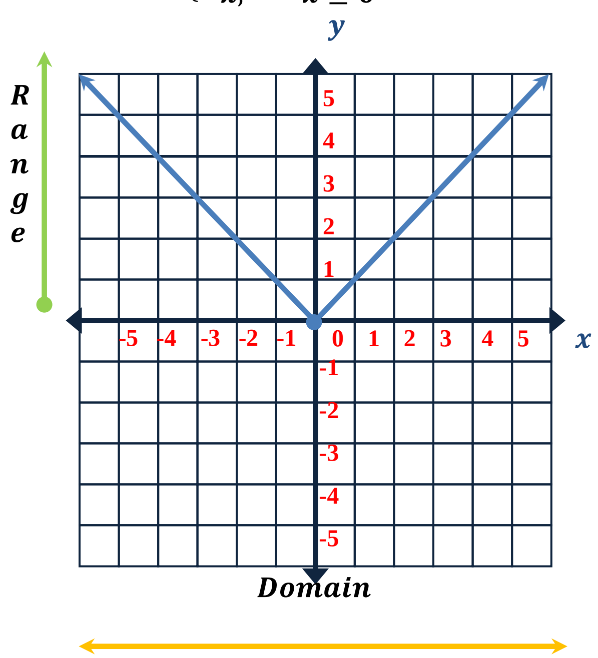
$To\ find\ f(3)\ use\ f(x) = -x^2$

$f(3) = -3^2$

$f(3) = -9$

Sample Problem 6: Find the domain and range of each function. Write in interval notation.

a. $f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$



$f(x) = -x, \quad x < 0$

$D_1 = (-\infty, 0)$

$f(x) = x, \quad x \geq 0$

$D_2 = [0, \infty)$

$D = D_1 \cup D_2 = (-\infty, \infty)$

$R = [0, \infty)$

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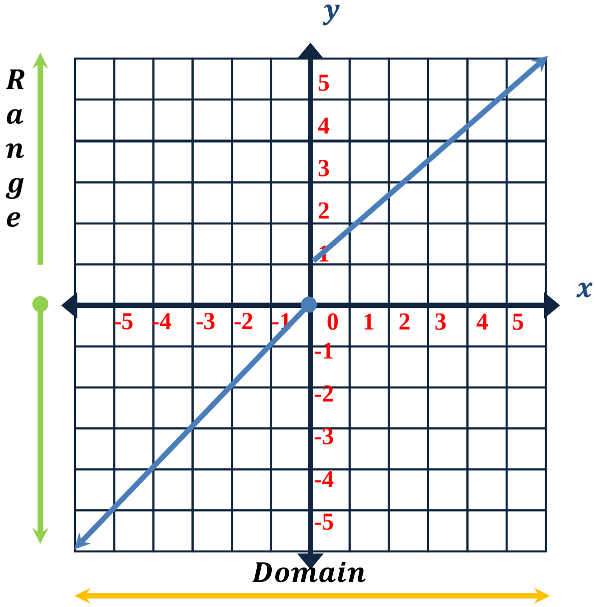
b. $f(x) = \begin{cases} x, & x \leq 0 \\ x+1, & x > 0 \end{cases}$



$f(x) = x, \quad x \leq 0$

$D_1 = (-\infty, 0]$

$f(x) = x + 1, \quad x > 0$

$D_2 = (0, \infty)$

$D = D_1 \cup D_2 = (-\infty, \infty)$

$R = (-\infty, 0] \cup (1, \infty)$